Table 1: A simple string subject to different excitations

Mathematical model	Physical model	Engineering applications
$-EA\frac{\partial^2 u}{\partial x^2}+\rho u_{tt}=mg\delta(x-vt)$	moving load	high speed track on cable
$u(0,t)=u(l,t)=0$ $u(x,0)=0,\dot{u}(x,0)=0$		
$-EA\frac{\partial^2 u}{\partial x^2}+\rho u_{tt}=0$	support excitations	cable on two mving mountains
$u(0,t)=a(t),u(l,t)=b(t)$ $u(x,0)=0,\dot{u}(x,0)=0$		
$-EA\frac{\partial^2 u}{\partial x^2}+\rho u_{tt}=P\delta(x-s)e^{i\omega t}$	forced vibration	cable detection by shaker
$u(0,t)=u(l,t)=0$ $u(x,0)=0,\dot{u}(x,0)=0$		
$-EA\frac{\partial^2 u}{\partial x^2}+\rho u_{tt}=0$	free vibration	ambient testing for cable
$u(0,t)=u(l,t)=0$ $u(x,0)=\phi(x),\dot{u}(x,0)=\psi(x)$		

Table 2: A simple beam subject to different excitations

Mathematical model	Physical model	Engineering applications
$EI\frac{\partial^4 u}{\partial x^4}+\rho u_{tt}=mg\delta(x-vt)$	moving load	high speed rail way across bridge
$u(0,t)=u(l,t)=0$ $u"(0,t)=u"(l,t)=0$ $u(x,0)=0,\dot{u}(x,0)=0$		
$EI\frac{\partial^4 u}{\partial x^4}+\rho u_{tt}=0$	support excitations	earthquake engineering
$u(0,t)=a(t),u(l,t)=b(t)$ $u"(0,t)=u"(l,t)=0$ $u(x,0)=0,\dot{u}(x,0)=0$		
$EI\frac{\partial^4 u}{\partial x^4}+\rho u_{tt}=P\delta(x-s)e^{i\omega t}$	forced vibration	bridge detection by shaker
$u(0,t)=u(l,t)=0$ $u"(0,t)=u"(l,t)=0$ $u(x,0)=0,\dot{u}(x,0)=0$		
$EI\frac{\partial^4 u}{\partial x^4}+\rho u_{tt}=0$	free vibration	ambient testing for bridge
$u(0,t)=u(l,t)=0$ $u"(0,t)=u"(l,t)=0$ $u(x,0)=\phi(x),\dot{u}(x,0)=\psi(x)$		