

D'Alembert's solution

I. Governing equation

$$u_{xx} = c^2 u_{tt}, \quad -\infty < x < \infty, \quad t > 0$$

II. Cauchy data

$$u(x, 0) = \phi(x),$$

$$\dot{u}(x, 0) = \psi(x),$$

III. D'Alembert's solution :

$$u(x, t) = \frac{1}{2}\phi(x+ct) + \frac{1}{2}\phi(x-ct) + \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(x) dx$$

VI. Methods for D'Alembert's solution :

1. change variables
2. Laplace transform
3. Fourier transform
4. Green's function

V. Domain of influence

IV. Domain of dependence

IIIV. Diamond rule:

$$u_A + u_C = u_B + u_D$$