Solve PDE by Taylor's expansion

I. For simplicity, we set

$$au_{xx}+2bu_{xy}+cu_{yy}=t(x,y)$$

II. Differentiating with respect to x, we have

$$au_{xxx}+2bu_{xxy}+cu_{xyy}+a_xu_{xx}+2b_xu_{xy}+c_xu_{yy}=t_x(x,y)$$

III. Differentiating with respect to y, we have

$$au_{xxy}+2bu_{xyy}+cu_{yyy}+a_yu_{xx}+2b_yu_{xy}+c_yu_{yy}=t_y(x,y)$$

VI. Cauchy data

$$u(f(s),g(s))=h(s)$$

$$u_x(f(s),g(s))=l(s)$$

$$u_y(f(s),g(s))=m(s)=\frac{h'(s)-l(s)f'(s)}{g'(s)}$$

V. Differentiating u_x with respect to x, we have

$$u_{xx}f'(s)+u_{xy}g'(s)=l'(s)$$

Differentiating with respect to x again, we have

$$[u_{xxx}f'(s)+u_{xxy}g'(s)]f'(s)+[u_{xxy}f'(s)+u_{xyy}g'(s)]g'(s)+u_{xx}f''(s)+u_{xy}g''(s)=l''(s)$$

The above equation can be reduced to

$$u_{xxx}f'^2(s)+2u_{xxy}f'(s)g'(s)+u_{xyy}g'^2(s)=l''(s)-u_{xx}f''(s)-u_{xy}g''(s)$$

Similarly, we have

$$u_{yxx}f'^2(s)+2u_{yyx}f'(s)g'(s)+u_{yyy}g'^2(s)=m''(s)-u_{xy}f''(s)-u_{yy}g''(s)$$

Reformulate in the matrix form for the unknow vector

$$det\begin{bmatrix} a(x,y) & b(x,y) & c(x,y) & 0 \\ 0 & a(x,y) & b(x,y) & c(x,y) \\ f'^2 & 2f'g' & g'^2 & 0 \\ 0 & f'^2 & 2f'g' & g'^2 \end{bmatrix}=0$$

Find the solvability condition for the unknown vector

$$\begin{bmatrix} u_{xxx} \\ u_{xxy} \\ u_{xyy} \\ u_{yyy} \end{bmatrix}$$

$$u(x,y)=u(x(s),y(s))+u_x(x-x(s))+u_y(x(s),y(s))(y-y(s))+\cdots$$