

國立臺灣海洋大學河海工程學系 2002 工程數學 (四) 第一次作業小考 參考解答
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1. Solve the PDE $u_x - u_y = 0$ for $u(x, y)$, subject to $u(s, 0) = s^2$.

Sol.

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$du = u_x dx + u_y dy$$

$$0 = u_x - u_y$$

$$\begin{cases} dx = 1 \\ dy = -1 \\ du = 0 \end{cases} \Rightarrow \begin{cases} \frac{dx}{dt} = 1 \\ \frac{dy}{dt} = -1 \\ \frac{du}{dt} = 0 \end{cases} \Rightarrow \begin{cases} x = t + x_0 \\ y = -t + y_0 \\ u = u_0 \end{cases}$$

$$x_0 = s, y_0 = 0, u_0 = s^2,$$

$$\therefore \begin{cases} x = t + s \\ y = -t \\ u = s^2 \end{cases}, \text{ then } u(x, y) = x^2 + 2xy + y^2 = (x + y)^2$$

2. Solve the PDE $2u_x + 3u_y = 4$ for $u(x, y)$, subject to $u(2s, 3s) = 5$.

Sol.

$$du = u_x dx + u_y dy$$

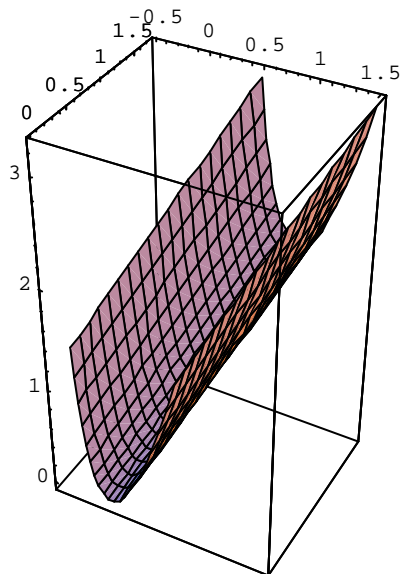
$$4 = u_x \cdot 2 - u_y \cdot 3$$

$$\begin{cases} dx = 2 \\ dy = 3 \\ du = 4 \end{cases} \Rightarrow \begin{cases} \frac{dx}{dt} = 2 \\ \frac{dy}{dt} = 3 \\ \frac{du}{dt} = 4 \end{cases} \Rightarrow \begin{cases} x = 2t + x_0 \\ y = 3t + y_0 \\ u = 4t + u_0 \end{cases}$$

$$x_0 = 2s, y_0 = 3s, u_0 = 5,$$

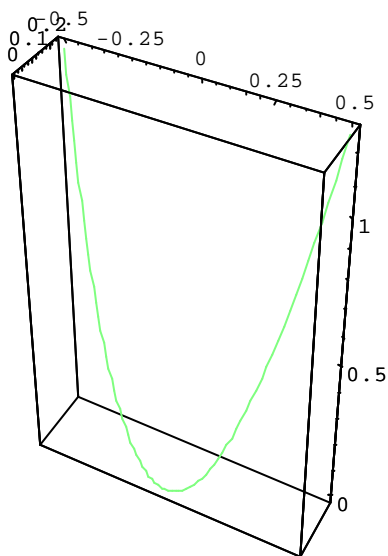
$$\therefore \begin{cases} x = 2t + 2s = 2(t + s) \\ y = 3t + 3s = 3(t + s) \\ u = 4t + 5 \end{cases}, u(x, y) = ?$$

```
f = ParametricPlot3D[92 t + s, 3 t + s^2, 4 t + 5 s^2 =,  
8 t, 0, 0.5 <, 8 s, -0.5, 0.5 <, AspectRatio @ 1.5 E
```



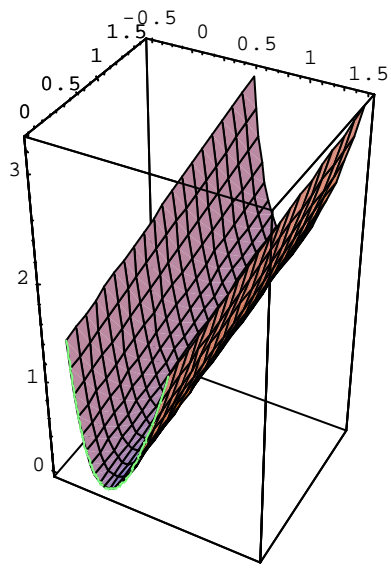
...Graphics3D...

```
g = ParametricPlot3D[9 s, s^2, 5 s^2, RGBColor@0.500008, 0.996109, 0.500008 D =,  
8 s, -0.5, 0.5 <, AspectRatio @ 1.5 E
```



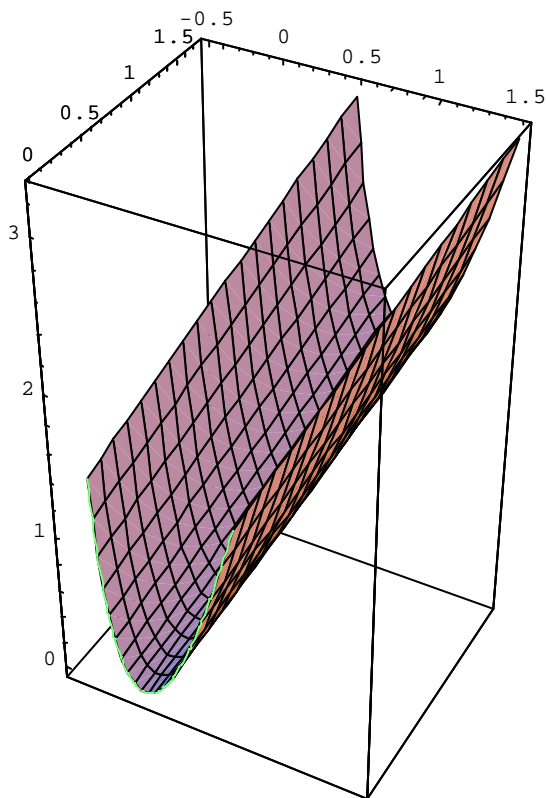
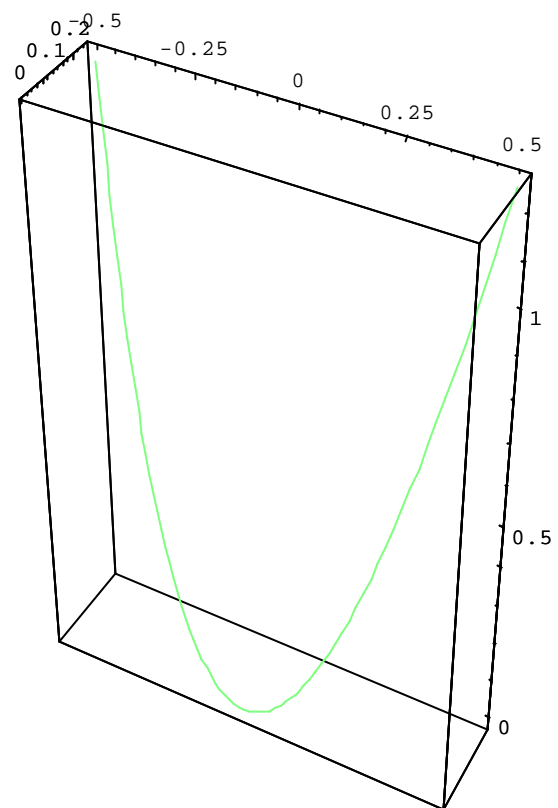
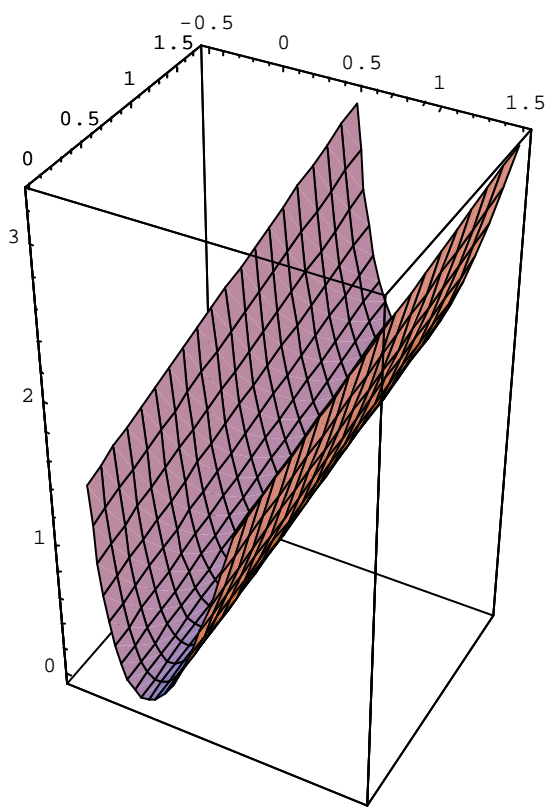
...Graphics3D...

```
h = Show@f, gD
```



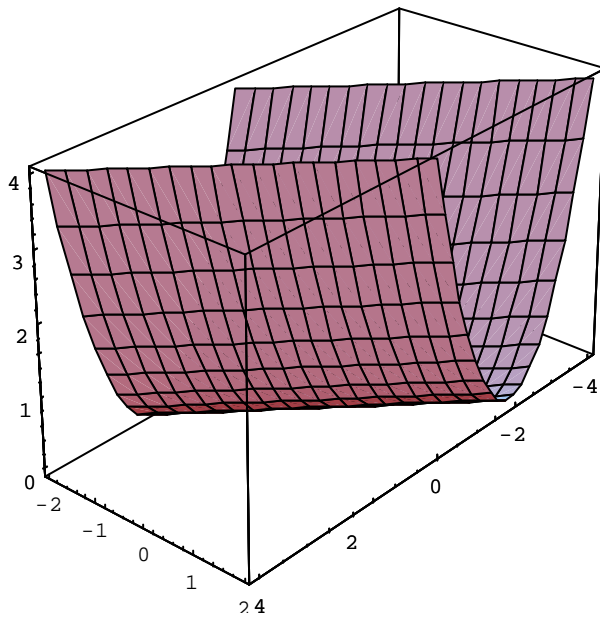
```
... Graphics3D ...
```

```
Show@GraphicsArray@88f, g<, 8h<<DD
```



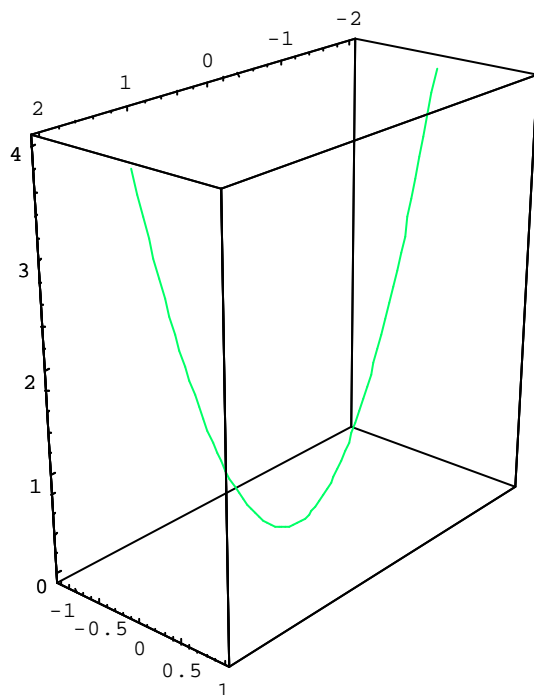
... GraphicsArray ...

```
f = ParametricPlot3D[9t + s, -t, s^2 =, 8t, -2, 2<,
  8s, -2, 2<, ViewPoint -> 82.750, 2.360, 1.540<, AspectRatio@ 1E
```



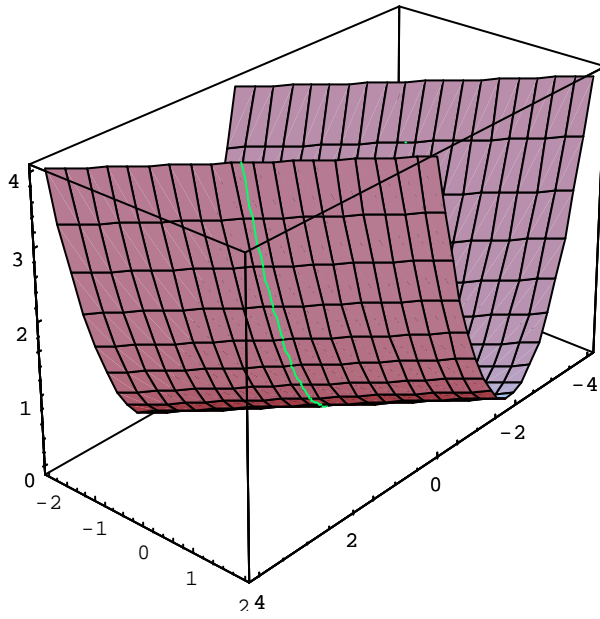
... Graphics3D ...

```
h = ParametricPlot3D[9s, 0, s^2, Hue@0.4D=,
  8s, -2, 2<, ViewPoint -> 82.750, 2.360, 1.540<E
```



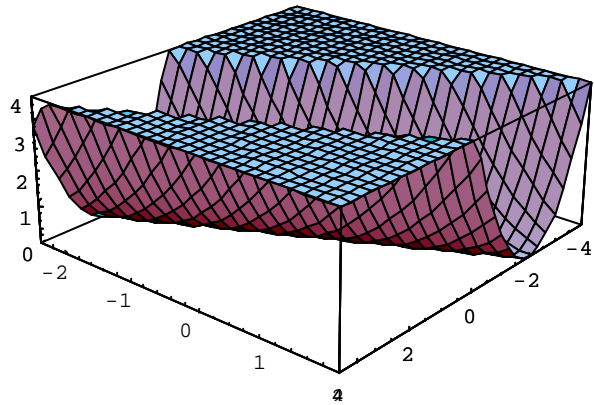
... Graphics3D ...

Show@f, hD



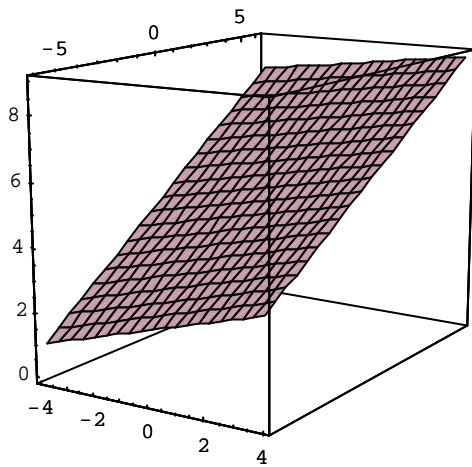
... Graphics3D ...

```
g = Plot3D[Ax2 + 2xy + y2, {x, -4, 4}, {y, -2.2, 2.2},  
PlotPoints @ 30, PlotRange @ 80, 4<, ViewPoint -> 82.750, 2.360, 1.540<E
```



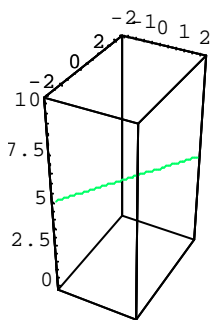
... SurfaceGraphics ...

```
f = ParametricPlot3D@82 t + 2 s, 3 t + 3 s, 4 t + 5 <,
  8 t, -1, 1 <, 8 s, -1, 1 <, ViewPoint -> 82.100, -3.270, 0.870 <D
```



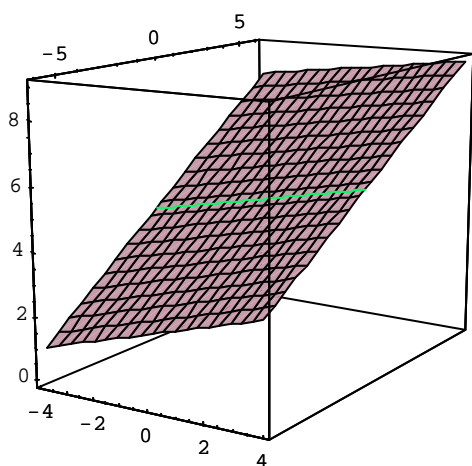
...Graphics3D...

```
g = ParametricPlot3D@82 s, 3 s, 5, Hue@0.4 <, 8 s, -1, 1 <D
```



...Graphics3D...

```
Show@f, g, ViewPoint -> 82.100, -3.270, 0.870 <D
```



...Graphics3D...