系級:_____ 姓名:_____ 學號:_____

國立臺灣海洋大學河海工程學系 2002 工程數學 (四) 第二次作業小考參考解答

1. G.E.: $x\,u_x + 2\,u_y = 2u$, I.C.: $u(s,0) = s^2\,g(s)$
Sol.

$dx\,u_x + dy\,u_y = du$

$x\,u_x + 2\,u_y = 2\,u$

$$\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = 2 \\ \frac{du}{dt} = 2u \end{cases} \Rightarrow \begin{cases} x = x_0\,e^t \\ y = 2t + y_0 \\ u = u_0\,e^{2t} \end{cases}$$

when $t = 0,\ (x,y,u) = (s,0,s^2\,g(s))$

$\therefore x_0 = s,\ y_0 = 0,\ u_0 = s^2\,g(s)$

$$\begin{cases} x = s\,e^t \\ y = 2t \\ u = s^2\,g(s)\,e^{2t} \end{cases}$$

$u(x,y) = x^2\,g(x\,e^{-\frac{y}{2}})$

2. G.E.: $x\,u_x = 2y\,u_y$, I.C.: (1) $u(1,y) = 2\,y + 1$, (2) $u(1,1) = 4$
Sol.
(法一):

$dx\,u_x = -dy\,u_y,\ -\dfrac{dx}{dy} = \dfrac{x}{2y},\ -\dfrac{2}{x}dx = \dfrac{1}{y}dy$

$x^2\,y = s,\ u(x,y) = f(s) = f(x^2\,y)$

(1). I.C.: $u(1,y) = 2\,y + 1$

$u(x,y) = 2(x^2\,y) + 1 = 2\,x^2\,y + 1$

(2). I.C.: $u(1,1) = 4$

$u(x,y) = 4(x^2\,y) = 4\,x^2\,y$

(法二)????:

$$\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = -2\,y \\ \frac{du}{dt} = 0 \end{cases} \Rightarrow \begin{cases} x = x_0\,e^t \\ y = y_0\,e^{-2t} \\ u = u_0 \end{cases},$$

when $t = 0,\ x_0 = 1,\ \ y_0 = s,\ \ , u_0 = 2s + 1.$

$$\begin{cases} x = e^t \\ y = s\,e^{-2t} \\ u = 2s + 1 \end{cases}, \ s = y\,e^{2t} = y\,x^2$$

$\therefore u = 2(y\,x^2) + 1 = 2y\,x^2 + 1$

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3. G.E.: $y\,u_x - x\,u_y = 3\,x$, I.C.: (1) $u(x,0) = x^2$, (2) $u(1,0) = 2$
Sol.

$$\begin{cases} \frac{dx}{dt} = y & (1) \\ \frac{dy}{dt} = -x & (2) \\ \frac{du}{dt} = 3x & (3) \end{cases}$$

由 (1),(2) 兩式, $x^2 + y^2 = s$

由 (2),(3) 兩式, $u = -3y + f(s) = -3y + f(x^2 + y^2)$

(一) I.C.: $u(x,0) = x^2$

use $f(s) = x^2 + y^2$, $u(x,y) = x^2 + y^2 - 3y$.

(二) I.C.: $u(1,0) = 2$

use $f(s) = 2(x^2 + y^2)$, $u(x,y) = 2\,x^2 + 2\,y^2 - 3y$.