$1.\ G.E.: u_x = u_y, I.C.: x_o(s) = s, y_0(s) = s, u_o(s) = 2s$

<sol>

$1.\ F(x, y, u, u_x, u_y) = u_x - u_y = 0, \quad x_o(s) = s, y_0(s) = s, u_o(s) = 2s \rightarrow$ General Form

2.令 $p = u_x, q = u_y$,則 $F_p = \dfrac{\partial F}{\partial p} = 1, F_q = \dfrac{\partial F}{\partial q} = -1, x\grave{}_0(s) = 1, y\grave{}_0(s) = 1$

$$\begin{vmatrix} F_p & F_q \\ x\grave{}_o(s) & y\grave{}_0(s) \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2 \neq 0$$

3.determine $p_0(s), q_0(s)$

由 $F(x_0(s), y_0(s), u_0(s), p_0(s), q_0(s)) = p_0(s) - q_0(s) = 0,$

$u\grave{}_0(s) = u_x x\grave{}_0(s) + u_y y\grave{}_0(s) = p_0(s) x\grave{}_0(s) + q_0(s) y\grave{}_0(s) \rightarrow 2 = p_0(s) + q_0(s)$

聯立得 $p_0(s) = 1, q_0(s) = 1$

4. 由 $F(x, y, u, p, q) = p - q = 0$,得

$$\frac{dx}{dt} = F_p, x(0,s) = x_0(s) \rightarrow \frac{dx}{dt} = F_p = 1, x_0(s) = s$$

$$\frac{dy}{dt} = F_q, y(0,s) = y_0(s) \rightarrow \frac{dy}{dt} = F_q = -1, y_0(s) = s$$

$$\frac{du}{dt} = pF_p + qF_q, u(0,s) = u_0(s) \rightarrow \frac{du}{dt} = p \times 1 + q \times -1 = p - q = 0, u_0(s) = 2s$$

$$\frac{dp}{dt} = -F_x - pF_u, p(0,s) = p_0(s) \rightarrow \frac{dp}{dt} = -0 - p \times 0, p_0(s) = 1$$

$$\frac{dq}{dt} = -F_y - qF_u, q(0,s) = q_0(s) \rightarrow \frac{dq}{dt} = -0 - q \times 0, q_0(s) = 1$$

得

$x(t,s) = t + s$

$y((t,s) = -t + s$

$u(t,s) = 2s$

$p(t,s) = 1$

$q(t,s) = 1$

聯立解得 u(x,y)=x+y

Ans:u(x,y)=x+y