系級:_____ 姓名:_____ 學號:_____

國立臺灣海洋大學河海工程學系 2002 工程數學 (四) 第三次作業小考參考解答

1. G.E.: $u_x\,u_y = 1$, I.C.: $u(s,s) = 2.5\,s$
Sol.

$$F(x,y,u,p,q) = u_x\,u_y - 1 = p\,q - 1$$

$$x_0 = s,\ y_0 = s\ u_0 = 2.5\,s$$

let $p = \lambda,\ q = \frac{1}{\lambda}$

$$\left\{\begin{array}{rcl} u - u_0 & = & p(\lambda)\,(x - x_0) + q(\lambda)\,(y - y_0) \\ 0 & = & p'(\lambda)\,(x - x_0) + q'(\lambda)\,(y - y_0) \end{array}\right. \Rightarrow \left\{\begin{array}{rcl} u - 2.5\,s & = & \lambda\,(x - s) + \frac{1}{\lambda}\,(y - s) \\ 0 & = & (x - s) - \frac{1}{\lambda^2}\,(y - s) \end{array}\right.$$

消去λ, 得 Monge Cone:

$$4\,(x - s)\,(y - s) = (u - 2.5\,s)^2$$

求 $p_0,\ \ q_0$:

$$\left\{\begin{array}{l} p_0\,q_0 - 1 = 0 \\ u_0'(s) = p_0(s)\,x_0'(s) + q_0\,y_0'(s) \end{array}\right. \Rightarrow \left\{\begin{array}{l} p_0\,q_0 - 1 = 0 \\ 2.5 = p_0(s) + q_0(s) \end{array}\right.$$

use $p_0 = 0.5,\ \ q_0 = 2$

$$F = pq - 1,\ F_x = 0,\ F_y = 0,\ F_u = 0,\ F_p = q,\ F_q = p$$

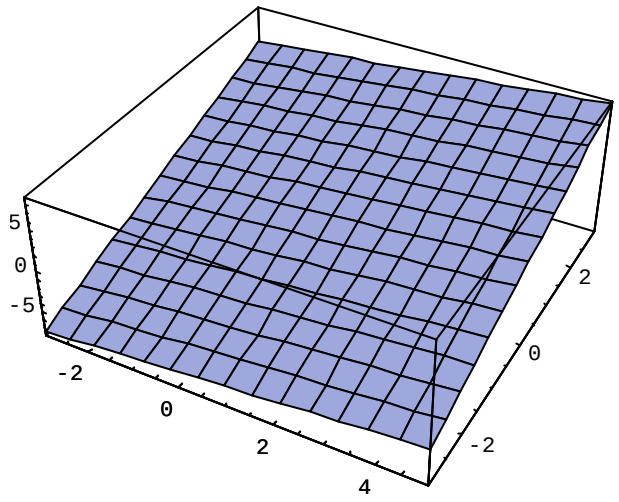
check $\left|\begin{array}{cc} F_p & F_q \\ x_0'(s) & y_0'(s) \end{array}\right| = \left|\begin{array}{cc} q & p \\ 1 & 1 \end{array}\right| \neq 0$

$$\left\{\begin{array}{rcccl} \frac{dx}{dt} & = & F_p & = & q \\ \frac{dy}{dt} & = & F_q & = & p \\ \frac{du}{dt} & = & p\,F_p + q\,F_q & = & 2\,p\,q \\ \frac{dp}{dt} & = & -F_x - p\,F_u & = & 0 \\ \frac{dq}{dt} & = & -F_y - q\,F_u & = & 0 \end{array}\right. \Rightarrow \left\{\begin{array}{rcl} x & = & 2\,t + s \\ y & = & 0.5\,t + s \\ u & = & 2\,t + 2.5\,s \\ p & = & 0.5 \\ q & = & 2 \end{array}\right.$$

$$\therefore u = \frac{1}{2}\,x + 2\,y$$

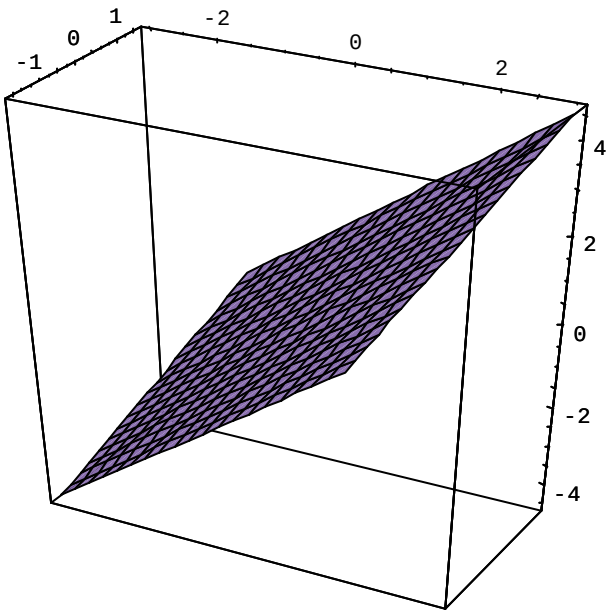
```
<< Graphics`ContourPlot3D`
```

```
a = Plot3D@0.5 x + 2 y, 8x, -3, 5<, 8y, -3, 3<D
```



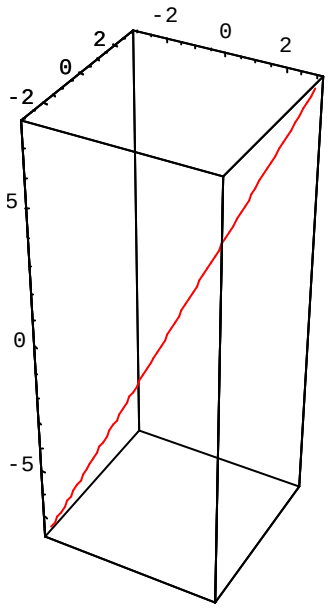
… SurfaceGraphics …

```
b = ParametricPlot3D@82 t + s, 0.5 t + s, 2 t + 2.5 s<, 8t, -1, 1<, 8s, -1, 1<, AspectRatio ® 1D
```



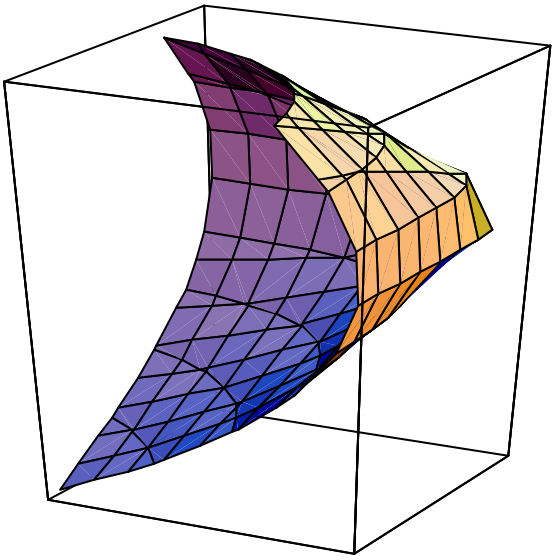
… Graphics3D …

```
c = ParametricPlot3D@8s, s, 2.5 s, Hue@1D<, 8s, -3, 3<D
```



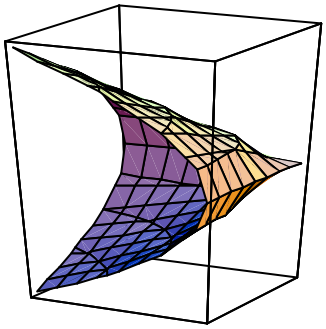
… Graphics3D …

```
d = ContourPlot3DAHx - 1L Hy - 1L - 1/4 Hu - 2.5L^2,
  8x, -1, 1<, 8y, -1, 1<, 8u, -1.5, 4.5<, AspectRatio ® 1E
```



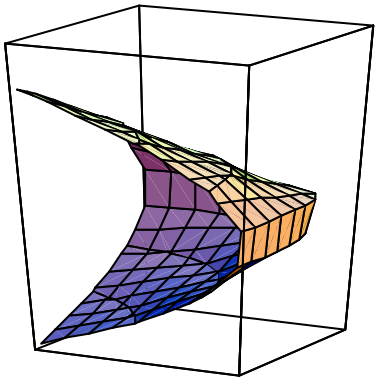
… Graphics3D …

```
e = ContourPlot3DAHxL HyL - 1/4 HuL^2, 8x, -2, 0<, 8y, -2, 0<, 8u, -4, 4<, AspectRatio ® 1E
```



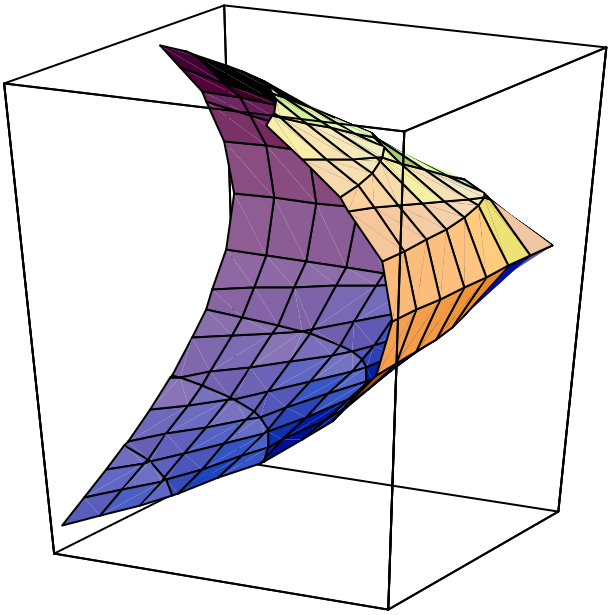
… Graphics3D …

```
f = ContourPlot3DAHx + 1L Hy + 1L - 1/4 Hu + 2.5L^2,
  8x, -2, -1<, 8y, -2, -1<, 8u, -4.5, 0<, AspectRatio ® 1E
```



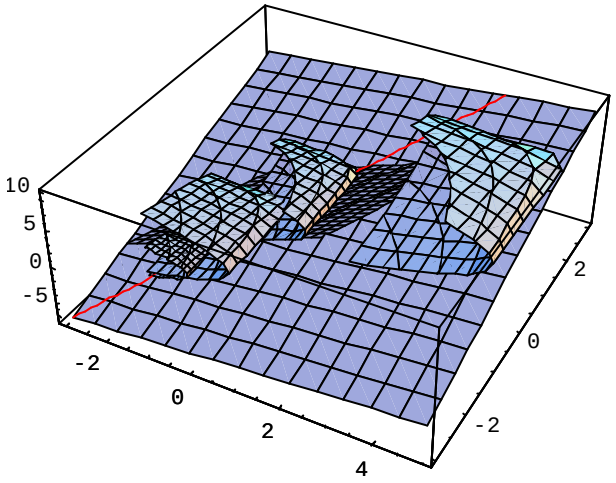
… Graphics3D …

```
g = ContourPlot3D[(x - 1 - 4L)(y - 1 - 1L - 1/4 (u - 2.5 - 4L)^2),
  {x, 2, 5}, {y, -1, 2}, {u, 0.5, 10}, AspectRatio → 1]
```

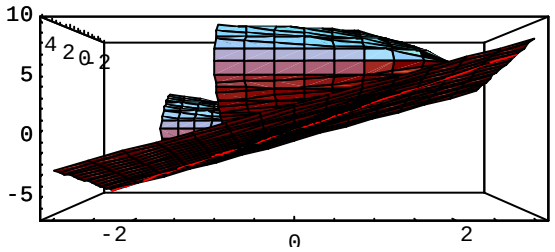


… Graphics3D …

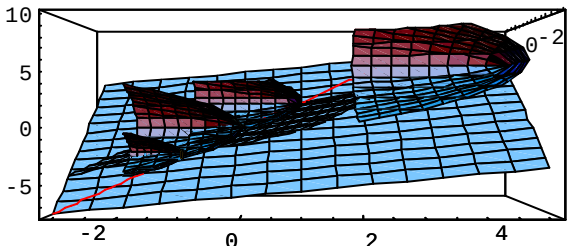
```
Show@a, b, c, d, e, f, gD
Show@%, ViewPoint -> 83.410, 0.000, 0.000<D
Show@%, ViewPoint -> 80.000, -4.000, 0.000<D
Show@%, ViewPoint -> 8-4.000, 0.000, 0.000<D
```



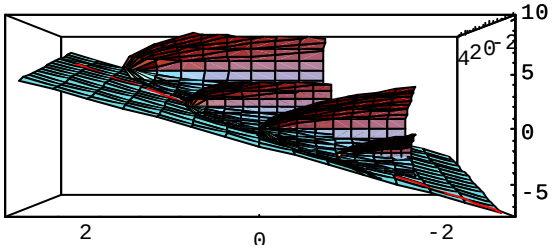
… Graphics3D …



… Graphics3D …



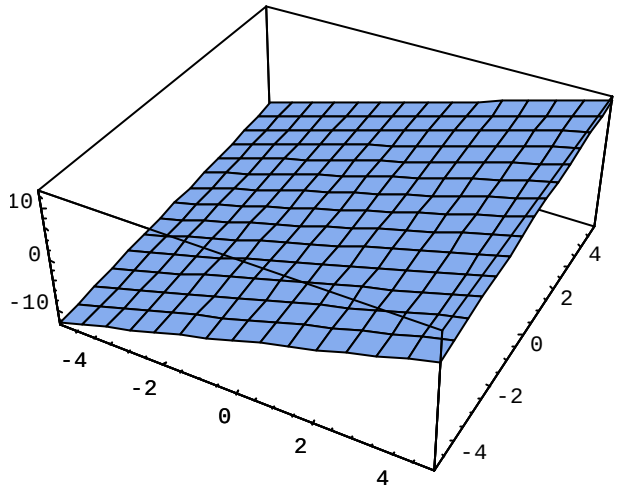
… Graphics3D …



… Graphics3D …

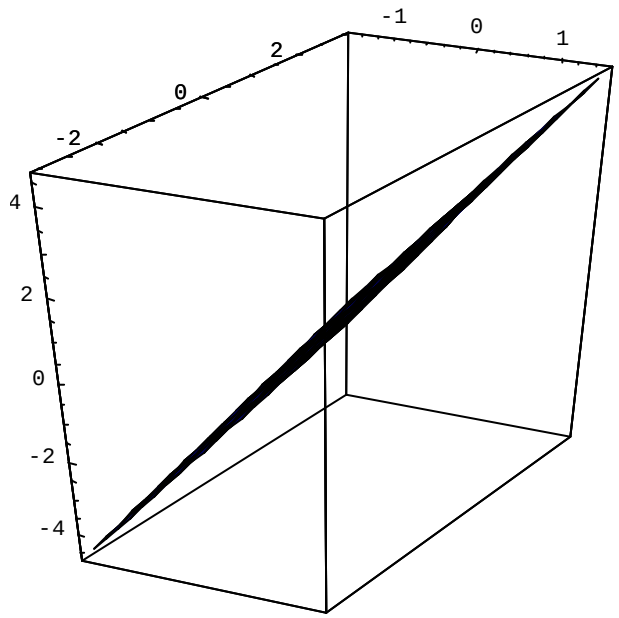
```
<< "Graphics`ContourPlot3D`"
```

```
a = Plot3D@2 x + 0.5 y, 8x, -5, 5<, 8y, -5, 5<D
```



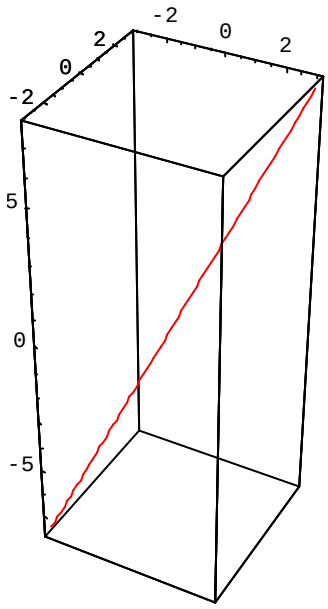
… SurfaceGraphics …

```
b = ParametricPlot3D@80.5 t + s, 2 t + s, 2 t + 2.5 s<, 8t, -1, 1<, 8s, -1, 1<, AspectRatio ® 1D
```



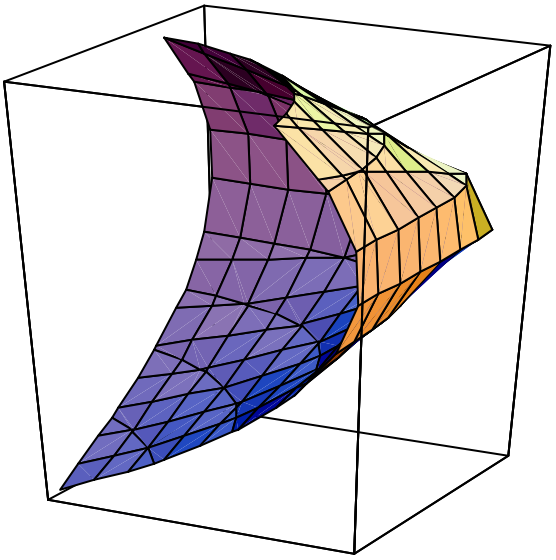
… Graphics3D …

```
c = ParametricPlot3D@8s, s, 2.5 s, Hue@1D<, 8s, -3, 3<D
```



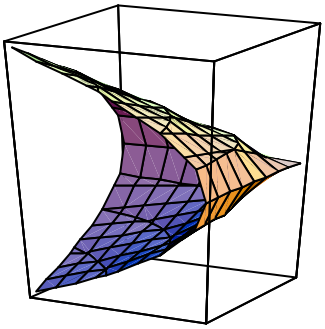
… Graphics3D …

```
d = ContourPlot3DAHx - 1L Hy - 1L - 1/4 Hu - 2.5L^2,
  8x, -1, 1<, 8y, -1, 1<, 8u, -1.5, 4.5<, AspectRatio ® 1E
```



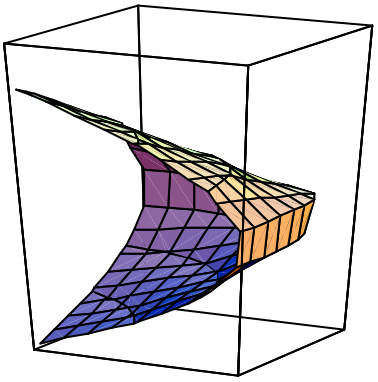
… Graphics3D …

```
e = ContourPlot3D[x, y - 1/4 u^2, {x, -2, 0}, {y, -2, 0}, {u, -4, 4}, AspectRatio -> 1]
```



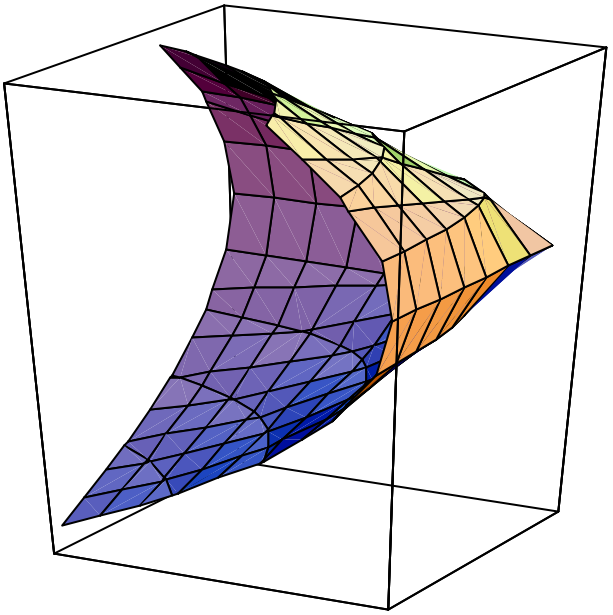
… Graphics3D …

```
f = ContourPlot3D[x + 1, y + 1, z - 1/4 (u + 2.5)^2,
  {x, -2, -1}, {y, -2, -1}, {u, -4.5, 0}, AspectRatio -> 1]
```



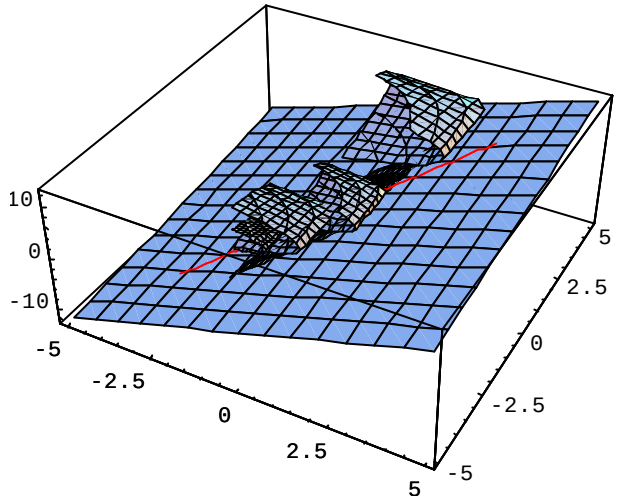
… Graphics3D …

```
g = ContourPlot3D[(x - 1)(y - 1 - 4)^(1/4) (u - 2.5 - 4)^2,
  {x, -1, 2}, {y, 2, 5}, {u, 0.5, 10}, AspectRatio → 1]
```

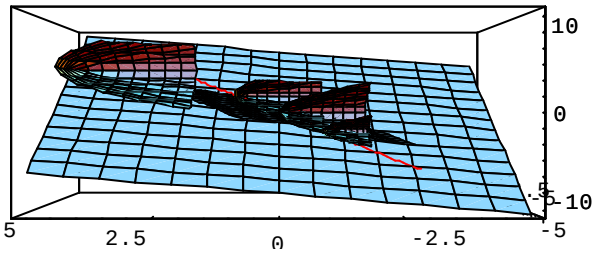


… Graphics3D …

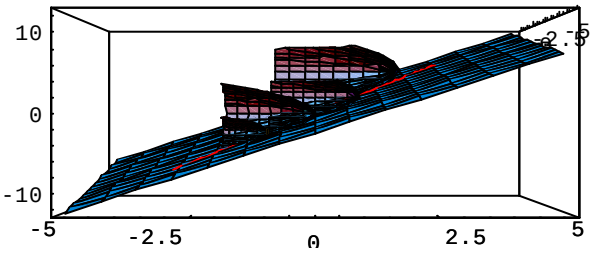
```
Show@a, b, c, d, e, f, gD
Show@%, ViewPoint -> 8-3.410, 0.000, 0.000<D
Show@%, ViewPoint -> 80.000, - 4.000, 0.000<D
```



… Graphics3D …



… Graphics3D …



… Graphics3D …