

題目:  $U_x U_y = 1$

解  $U = \alpha x + \frac{1}{\alpha} y + \beta$ , 其中  $\alpha, \beta$  為任意數

(1) 猜  $\beta = n\alpha, n$  為任意實數

$$U = (x+n)\alpha + \frac{1}{\alpha} y$$

$$0 = (x+n) - \frac{1}{\alpha^2} y$$

$$U = 2(x+n)\alpha \Rightarrow \alpha = \frac{U}{2(x+n)}$$

$$U^2 = 4(x+n)y$$

(2) 猜  $\beta = \frac{n}{\alpha}, n$  為任意實數

$$U = \alpha x + \frac{1}{\alpha}(y+n)$$

$$0 = x - \frac{1}{\alpha^2}(y+n)$$

$$U = 2\alpha x \Rightarrow \alpha = \frac{U}{2x}$$

$$U^2 = 4(y+n)x$$

(3) 猜  $\beta = 0$

$$U = \alpha x + \frac{1}{\alpha} y$$

$$0 = x - \frac{1}{\alpha^2} y$$

$$U = 2\alpha x \Rightarrow \alpha = \frac{U}{2x}$$

$$U^2 = 4xy$$

設 Cauchy Data 爲  $U(s, s) = 2s$ :

$$U' = p_0 x' + q_0 y' \Rightarrow \begin{aligned} p_0 q_0 - 1 &= 0 \\ 2 &= p_0 + q_0 \end{aligned}$$

$$p_0 = 1, q_0 = 1$$

$$\begin{vmatrix} F_p & F_q \\ x'(s) & y'(s) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

設 Cauchy Data 爲  $U(s, 4s) = 4s$ :

$$U' = p_0 x' + q_0 y' \Rightarrow \begin{aligned} p_0 q_0 - 1 &= 0 \\ 4 &= p_0 + 4q_0 \end{aligned}$$

$$p_0 = 2, q_0 = 0.5$$

$$\begin{vmatrix} F_p & F_q \\ x'(s) & y'(s) \end{vmatrix} = \begin{vmatrix} 0.5 & 2 \\ 1 & 4 \end{vmatrix} = 0$$

Singular Solution ?

題目:  $U_x U_y = 1$  ,  $U(s, s) = 2.5s$

解  $U = \alpha x + \frac{1}{\alpha}y + \beta$  , 並代入 Cauchy Data:

$$2.5s = s\alpha + \frac{1}{\alpha}s + \beta \quad (1)$$

$$2.5 = \alpha + \frac{1}{\alpha} \quad (2)$$

將 ( 1 - 2 ) 式代入 ( 1 - 1 ) 式, 可得

$$2.5s = 2.5s + \beta$$

即  $\beta = 0$  , 代入 ( 1 - 1 ) 式即可算出  $\alpha$  :

$$\left(\alpha + \frac{1}{\alpha}\right)s = 2.5s$$

$$\alpha^2 - 2.5\alpha + 1 = 0$$

$$(\alpha - 2)(2\alpha - 1) = 0$$

$$\alpha = 2 \text{ or } \alpha = \frac{1}{2}$$

代回原式則為

$$U = 2x + \frac{1}{2}y$$

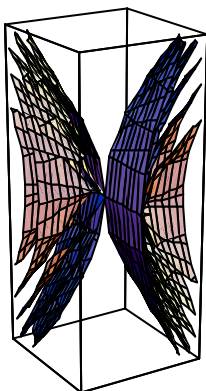
$$U = \frac{1}{2}x + 2y$$

```
<< "Graphics`ContourPlot3D`"

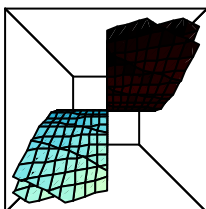
a1 = Table[ContourPlot3D[4 Hx + nL y - u2, {x, -5, 5}, {y, -5, 5}, {z, 0, 2},
  {u, -12, 12}, DisplayFunction -> Identity], {n, 1, 20}];

a2 = Table[ContourPlot3D[4 Hx + nL y - u2, {x, 0, 5}, {y, 0, 5}, {z, 0, 2},
  {u, -12, 12}, DisplayFunction -> Identity], {n, 1, 20}];
```

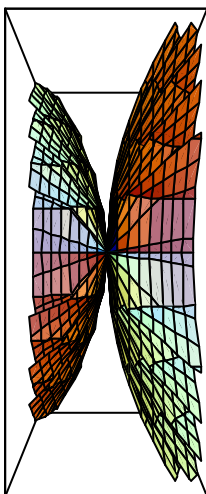
```
g = Show@a1, a2, ViewPoint -> 83.080, -1.910, 1.300<,
  DisplayFunction@ $DisplayFunctionD
Show@%, ViewPoint -> 80, 0.000, 1.000<D
Show@%, ViewPoint -> 81.000, 0.000, 0.000<D
```



... Graphics3D ...

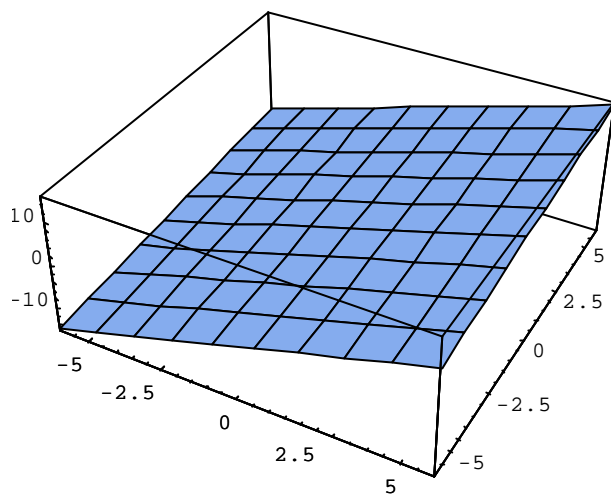


... Graphics3D ...



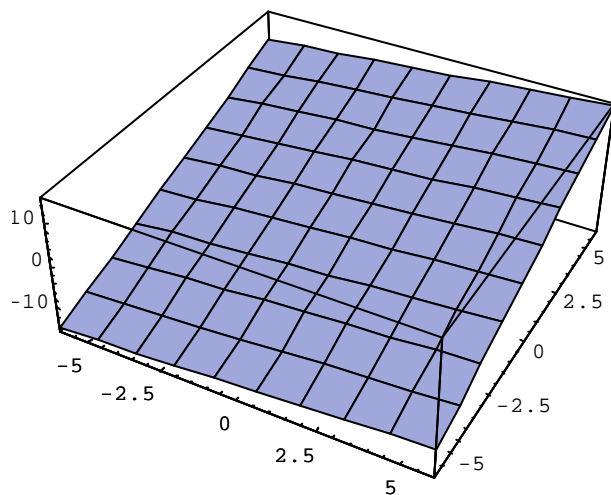
... Graphics3D ...

```
c = Plot3D[2 x + 0.5 y, {x, -6, 6}, {y, -6, 6}, PlotPoints -> 10]
```



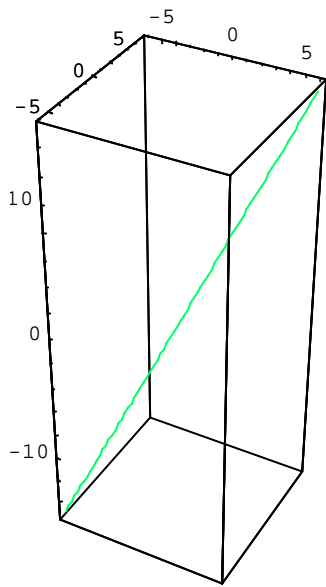
... SurfaceGraphics ...

```
d = Plot3D[0.5 x + 2 y, {x, -6, 6}, {y, -6, 6}, PlotPoints -> 10]
```



... SurfaceGraphics ...

```
e = ParametricPlot3D[8s, s, 2.5 s, Hue@0.4D<, 8s, - 6, 6<D
```



... Graphics3D ...