系級:_____ 姓名:_____ 學號:_____

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Given an infinite string, solve the solution.

$$\text{G.E.} \quad c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

$$\text{I.C.} \quad \begin{aligned} u(x,0) &= \phi(x) \\ \dot{u}(x,0) &= \psi(x) \end{aligned}$$

1. Verify D'Alembert's solution

$$u(x,t) = \frac{1}{2}\phi(x+ct) + \frac{1}{2}\phi(x-ct) + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(\tau) d\tau$$

satisfy PDE and I.C..

2. Derive the solution by yourself.
3. Plot

   (a) $\mathrm{X-Y}$ plot
   (b) $3-\mathrm{D}$ plot
   (c) Contour plot

where (1) $\begin{cases} u(x,0) = \phi(x) \\ \dot{u}(x,0) = 0 \end{cases}$, (2) $\begin{cases} u(x,0) = 0 \\ \dot{u}(x,0) = \psi(x) \end{cases}$

in which

$$\phi(x) = \begin{cases} 1, & -1 < x < 1 \\ 0, & otherwise, \end{cases} \quad \psi(x) = \begin{cases} 1, & -1 < x < 1 \\ 0, & otherwise \end{cases}$$