

國立臺灣海洋大學河海工程學系 2002 工程數學 (四) 第五次作業小考參考解答

1. Sol.

$$u(x, t) = \frac{1}{2}\phi(x+ct) + \frac{1}{2}\phi(x-ct) + \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(\tau) d\tau$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}\phi'(x+ct) + \frac{1}{2}\phi'(x-ct) + \frac{1}{2c}[\psi(x+ct) - \psi(x-ct)]$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\phi''(x+ct) + \frac{1}{2}\phi''(x-ct) + \frac{1}{2c}[\psi'(x+ct) - \psi'(x-ct)]$$

$$\frac{\partial u}{\partial t} = \frac{c}{2}\phi'(x+ct) - \frac{c}{2}\phi'(x-ct) + \frac{1}{2c}[c\psi(x+ct) + c\psi(x-ct)]$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{c^2}{2}\phi''(x+ct) + \frac{c^2}{2}\phi''(x-ct) + \frac{1}{2c}[c^2\psi'(x+ct) - c^2\psi'(x-ct)]$$

(1) 代入 PDE

$$\begin{aligned} c^2 \frac{\partial^2 u}{\partial x^2} &= c^2 \left[\frac{1}{2}\phi''(x+ct) + \frac{1}{2}\phi''(x-ct) + \frac{1}{2c}[\psi'(x+ct) - \psi'(x-ct)] \right] \\ &= \frac{c^2}{2}\phi''(x+ct) + \frac{c^2}{2}\phi''(x-ct) + \frac{1}{2c}[c^2\psi'(x+ct) - c^2\psi'(x-ct)] \\ &= \frac{\partial^2 u}{\partial t^2} \end{aligned}$$

$$\therefore c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

(2) 代入 I.C.

$$u(x, 0) = \frac{1}{2}\phi(x) + \frac{1}{2}\phi(x) + \frac{1}{2c} \int_x^x \psi(\tau) d\tau = \phi(x)$$

$$\dot{u}(x, 0) = \frac{c}{2}\phi'(x) - \frac{c}{2}\phi'(x) + \frac{1}{2c}[c\psi(x) + c\psi(x)] = \psi(x)$$

2. Sol.

$$\text{let } \begin{cases} u(x, t) = g(x+ct) + f(x-ct) \\ \dot{u}(x, t) = cg'(x+ct) - cf'(x-ct) \end{cases} \Rightarrow \begin{cases} u(x, 0) = g(x) + f(x) = \phi(x) & (1) \\ \dot{u}(x, 0) = cg'(x) - cf'(x) = \psi(x) & (2) \end{cases}$$

(2) 式對 x 積分

$$c \int_0^x g'(x) dx - c \int_0^x f'(x) dx = \int_0^x \psi(\tau) d\tau$$

$$g(x) - f(x) = \frac{1}{c} \int_0^x \psi(\tau) d\tau \quad (3)$$

(1)(3) 聯立

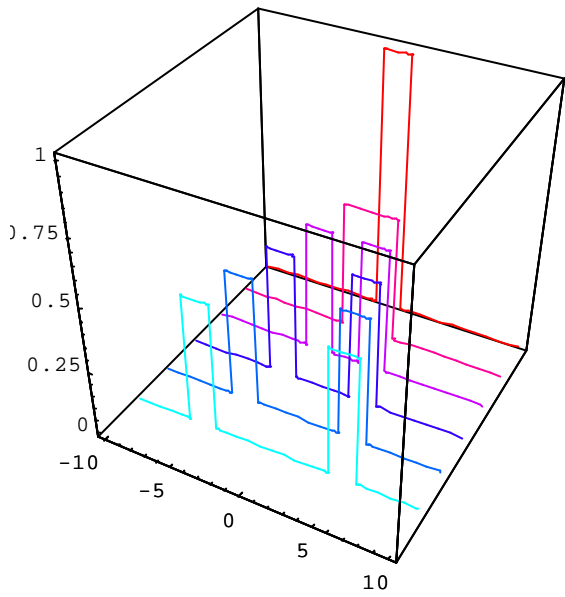
$$\begin{cases} g(x) + f(x) = \phi(x) \\ g(x) - f(x) = \frac{1}{c} \int_0^x \psi(\tau) d\tau \end{cases} \Rightarrow \begin{cases} g(x) = \frac{1}{2}\phi(x) + \frac{1}{2c} \int_0^x \psi(\tau) d\tau \\ f(x) = \frac{1}{2}\phi(x) - \frac{1}{2c} \int_0^x \psi(\tau) d\tau \end{cases}$$

$$\therefore \begin{cases} g(x+ct) = \frac{1}{2}\phi(x+ct) + \frac{1}{2c} \int_0^{x+ct} \psi(\tau) d\tau \\ f(x-ct) = \frac{1}{2}\phi(x-ct) - \frac{1}{2c} \int_0^{x-ct} \psi(\tau) d\tau \end{cases}$$

$$\begin{aligned} u(x,t) &= g(x+ct) + f(x-ct) \\ &= \frac{1}{2}\phi(x+ct) + \frac{1}{2}\phi(x-ct) + \frac{1}{2c} \left(\int_0^{x+ct} \psi(\tau) d\tau - \int_0^{x-ct} \psi(\tau) d\tau \right) \\ &= \frac{1}{2}\phi(x+ct) + \frac{1}{2}\phi(x-ct) + \frac{1}{2c} \left(\int_0^{x+ct} \psi(\tau) d\tau + \int_{x-ct}^0 \psi(\tau) d\tau \right) \\ &= \frac{1}{2}\phi(x+ct) + \frac{1}{2}\phi(x-ct) + \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(\tau) d\tau \end{aligned}$$

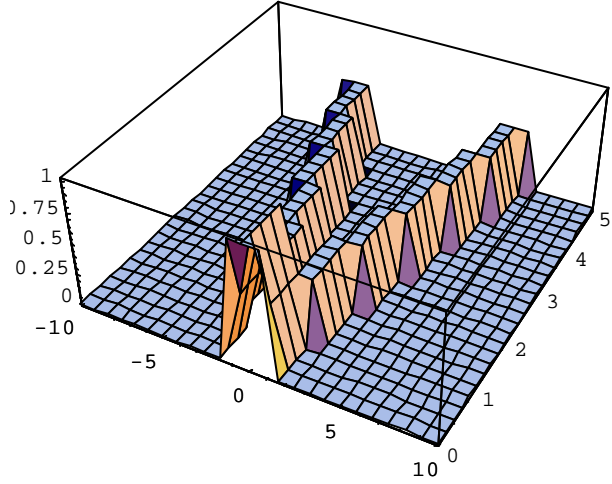
```
f[x_D := If[-1 < x < 1, 1, 0]
g = Table[Plot[Evaluate[ $\frac{1}{2} f[x + tD] + \frac{1}{2} f[x - tD]$ , 8x, -10, 10],
  DisplayFunction @ Identity, PlotStyle @ Hue[1 -  $\frac{t}{10}$ ], 8t, 5, 0, -1 < E
... Graphics ...
```

```
<< "Graphics`Graphics3D`"
Show@StackGraphics@gDD
```

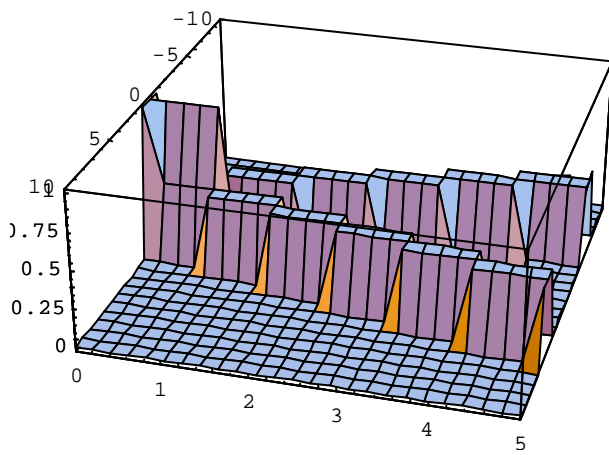


... Graphics3D ...

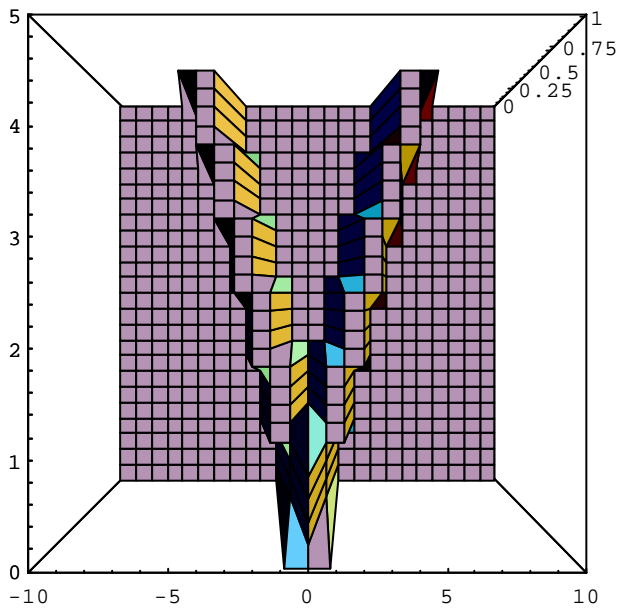
```
u = Plot3D[Evaluate[ $\frac{1}{2} f[x + tD] + \frac{1}{2} f[x - tD]$ , 8x, -10, 10], 8t, 0, 5, PlotPoints @ 25E
Show@%, ViewPoint -> 84.000, 1.140, 2.110 <D
Show@%, ViewPoint @ 80, 0, 1 <D
Show@%, ViewPoint @ 81, 0, 0 <D
Show@%, ViewPoint @ 80, 1, 0 <D
```



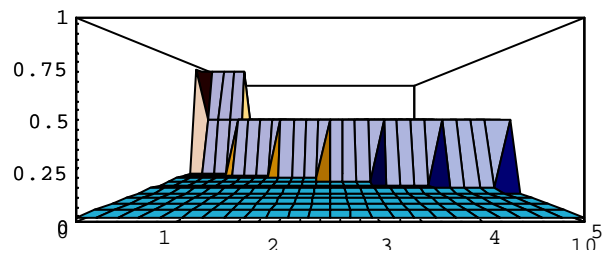
... SurfaceGraphics ...



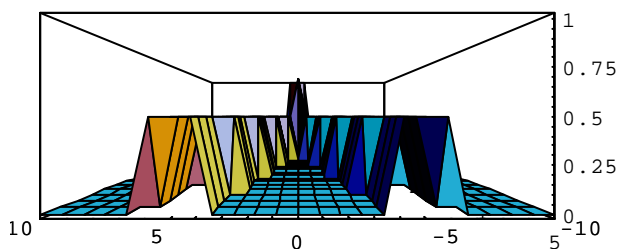
... SurfaceGraphics ...



... SurfaceGraphics ...

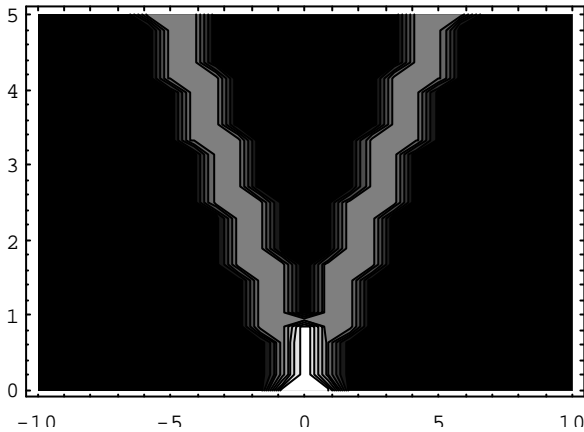


... SurfaceGraphics ...



... SurfaceGraphics ...

```
Show@ContourGraphics@uD, AspectRatio @ 0.7D
```



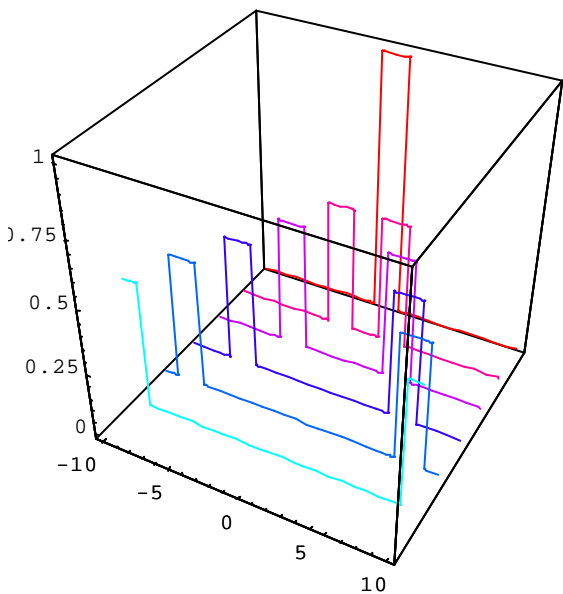
... ContourGraphics ...

```
g2 = Table[Plot[Evaluate[A  $\frac{1}{2}$  f[x + 2 tD] +  $\frac{1}{2}$  f[x - 2 tD], 8x, -10, 10<,
```

```
DisplayFunction @ Identity, PlotStyle @ Hue[A1 -  $\frac{t}{10}$ ], 8t, 5, 0, -1<E
```

... Graphics ... Graphics ... Graphics ... Graphics ... Graphics ... Graphics ...

```
Show@StackGraphics@g2DD
```



... Graphics3D ...

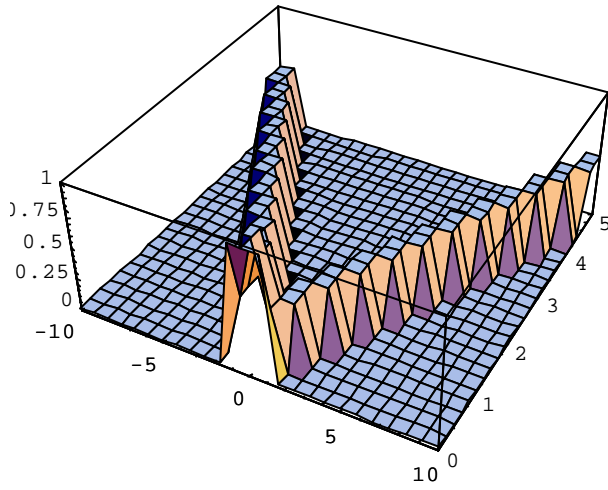
```
u2 = Plot3D[A  $\frac{1}{2}$  f[x + 2 tD] +  $\frac{1}{2}$  f[x - 2 tD], 8x, -10, 10<, 8t, 0, 5<, PlotPoints @ 25E
```

```
Show@%, ViewPoint -> 84.000, 1.140, 2.110<D
```

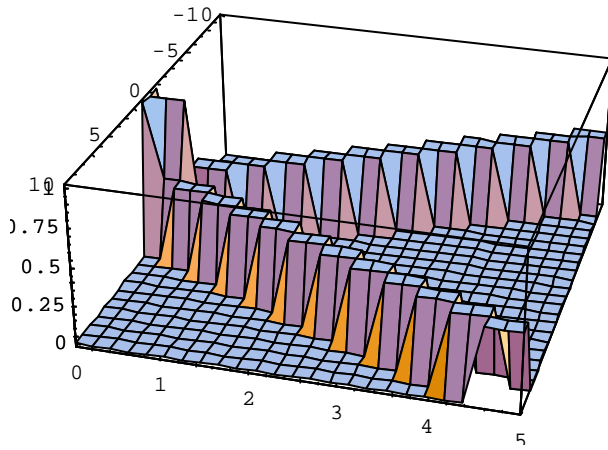
```
Show@%, ViewPoint @ 80, 0, 1<D
```

```
Show@%, ViewPoint @ 81, 0, 0<D
```

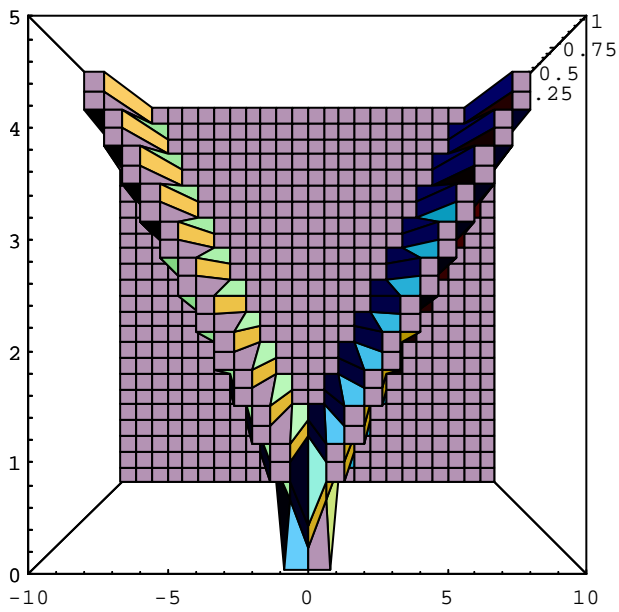
```
Show@%, ViewPoint @ 80, 1, 0<D
```



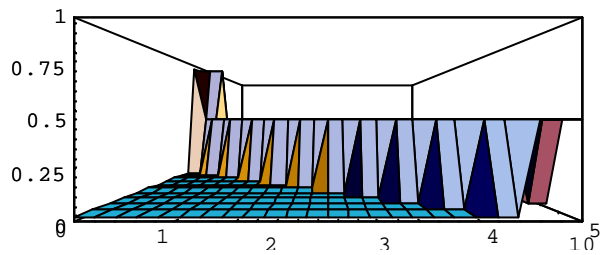
... SurfaceGraphics ...



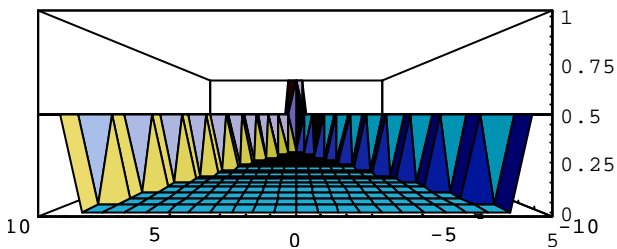
... SurfaceGraphics ...



... SurfaceGraphics ...

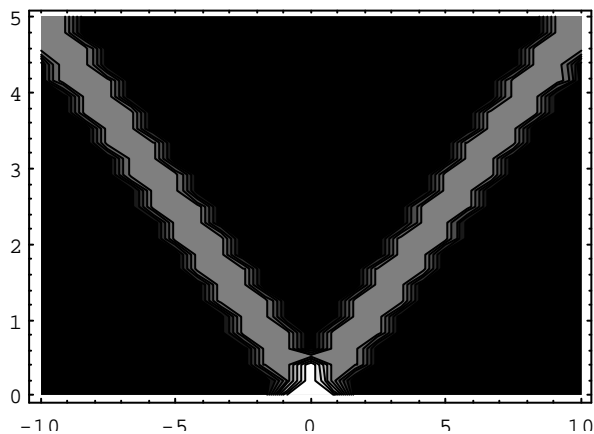


... SurfaceGraphics ...



... SurfaceGraphics ...

Show@ContourGraphics@u2D, AspectRatio@0.7D



... ContourGraphics ...

```

y@x_D := If[-1 < x < 1, 1, 0]

g = Table[Plot[Evaluate[ $\frac{1}{2} e^{x+t} - \frac{1}{2} e^{x-t}$ ], {x, -10, 10}], {t, 0, 5, 0.1}]

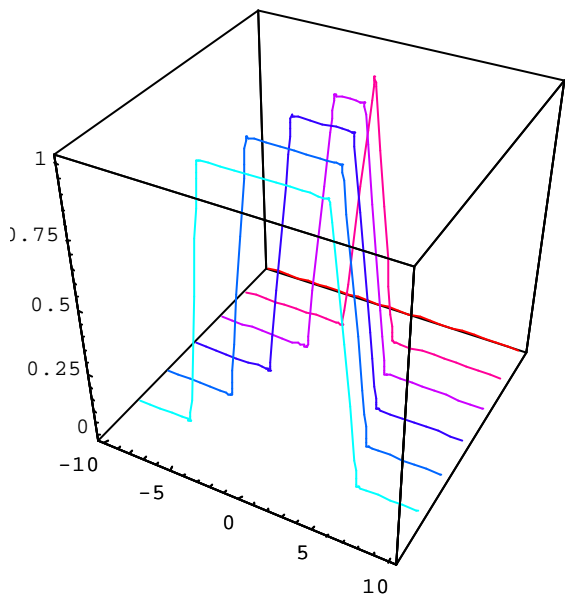
DisplayFunction @ Identity, PlotStyle @ Hue[1 -  $\frac{t}{10}$ ], 8t, 5, 0, -1 < E

8...Graphics ..., ...Graphics ..., ...Graphics ..., ...Graphics ..., ...Graphics ..., ...Graphics ...<

<< "Graphics`Graphics3D`"

Show@StackGraphics@gDD

```



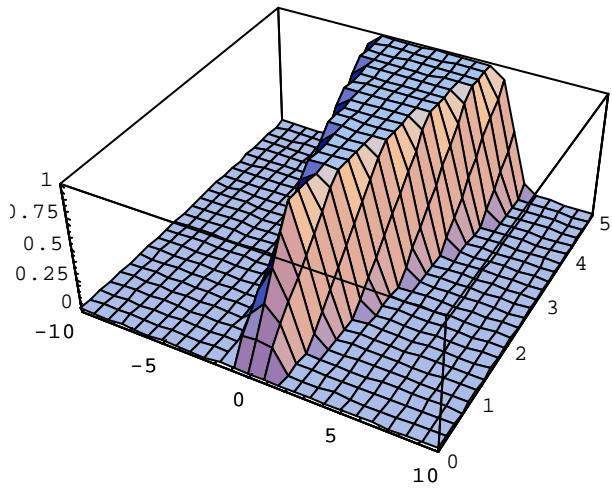
...Graphics3D...

```

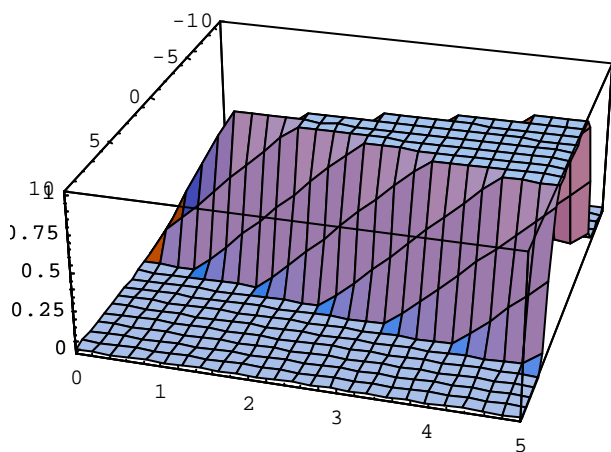
u = Plot3D[Evaluate[ $\frac{1}{2} e^{x+t} - \frac{1}{2} e^{x-t}$ ], {x, -10, 10}, {t, 0, 5}, PlotPoints @ 25E

Show@%, ViewPoint -> 84.000, 1.140, 2.110 < D
Show@%, ViewPoint @ 80, 0, 1 < D
Show@%, ViewPoint @ 81, 0, 0 < D
Show@%, ViewPoint @ 80, -1, 0 < D

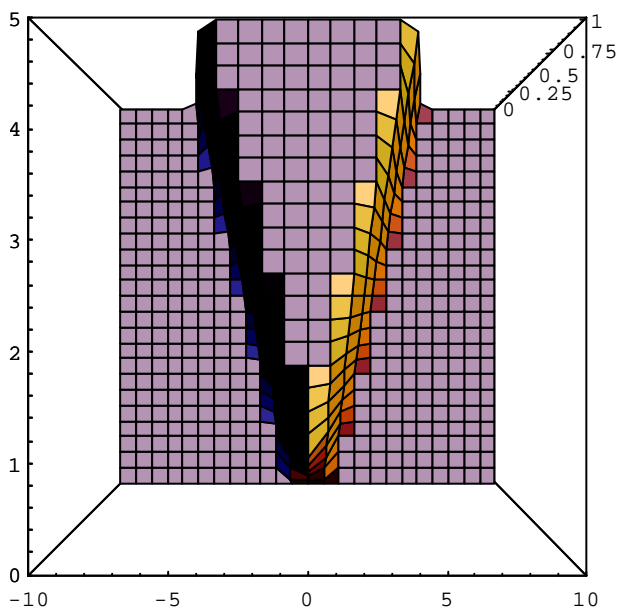
```



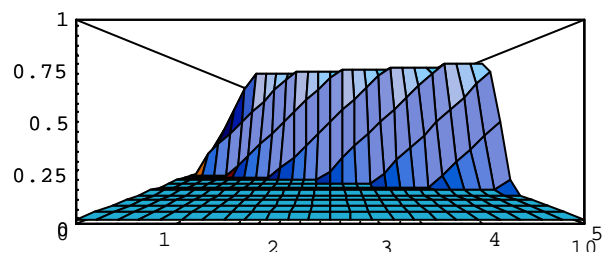
...SurfaceGraphics...



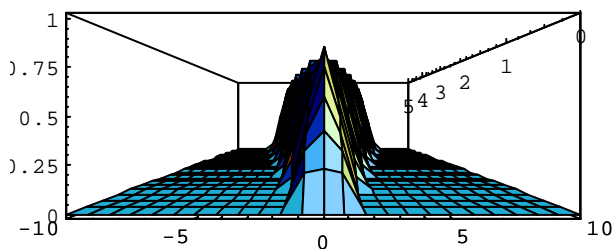
...SurfaceGraphics...



...SurfaceGraphics...

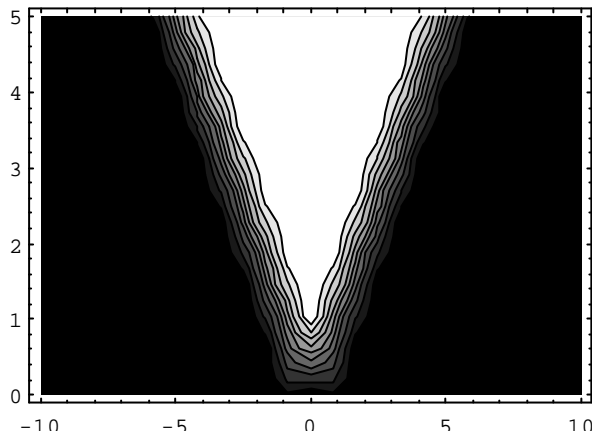


...SurfaceGraphics...



... SurfaceGraphics ...

Show@ContourGraphics@uD, AspectRatio@0.7D



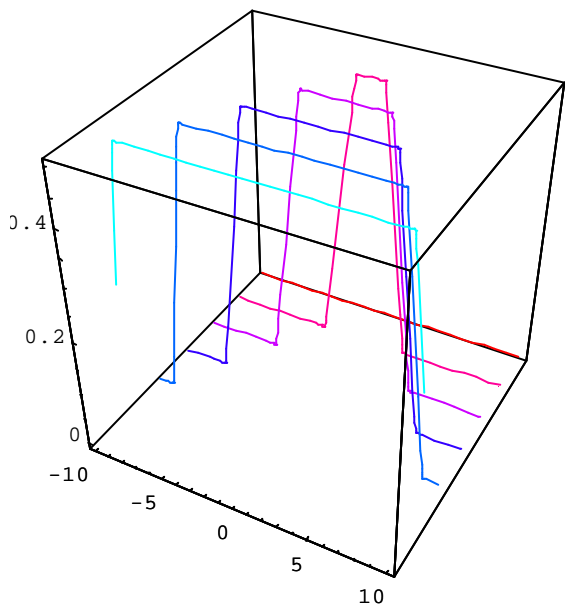
... ContourGraphics ...

g2 = Table@Plot@Evaluate@ $\frac{1}{4} \frac{x+2t}{x-2t} \text{Hy@tDL} \hat{a} tE, 8x, -10, 10<,$

DisplayFunction@Identity, PlotStyle@Hue@1 - $\frac{t}{10}$, 8t, 5, 0, -1<E

8... Graphics ..., ... Graphics ..., ... Graphics ..., ... Graphics ..., ... Graphics ..., ... Graphics ...<

Show@StackGraphics@g2DD



... Graphics3D ...

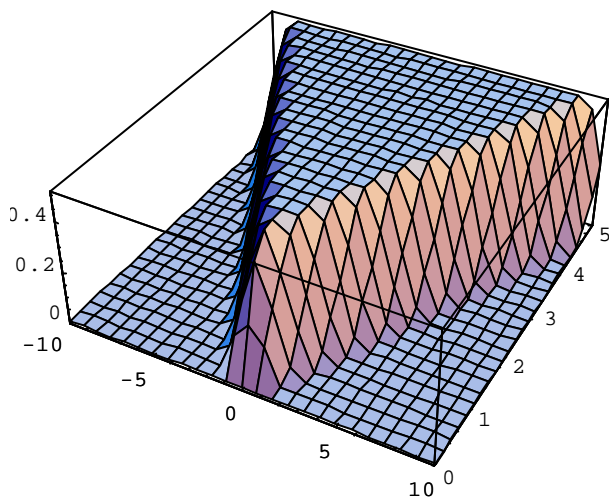
u2 = Plot3D@Evaluate@ $\frac{1}{2} \frac{x+2t}{x-2t} \text{Hy@tDL} \hat{a} tE, 8x, -10, 10<, 8t, 0, 5<, PlotPoints@25E$

Show@%, ViewPoint -> 84.000, 1.140, 2.110<D

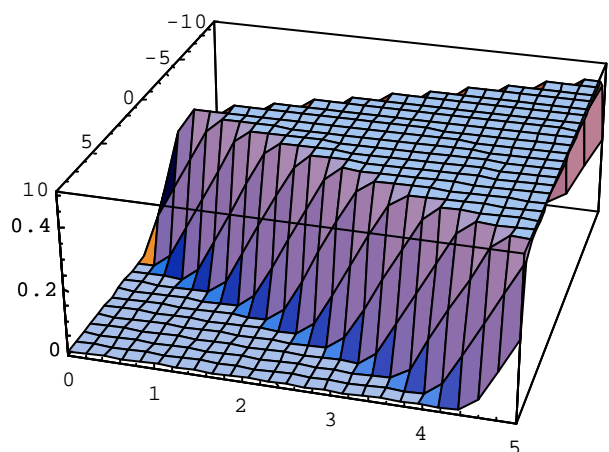
Show@%, ViewPoint@80, 0, 1<D

Show@%, ViewPoint@81, 0, 0<D

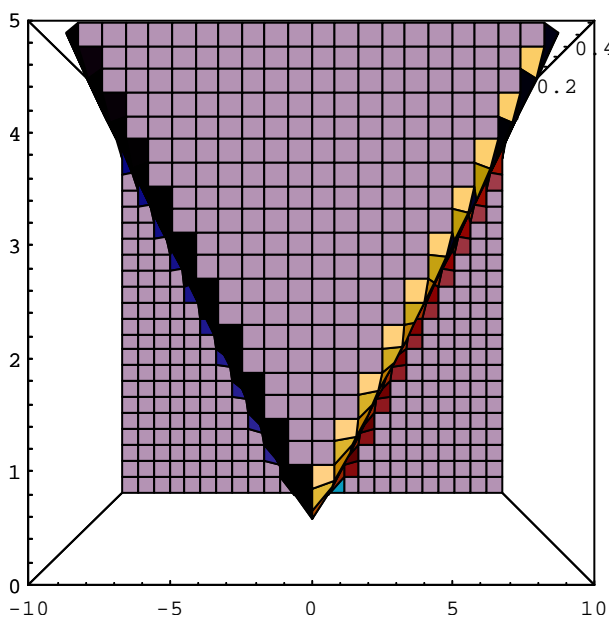
Show@%, ViewPoint@80, -1, 0<D



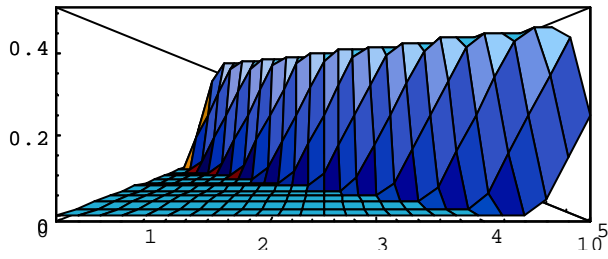
...SurfaceGraphics ...



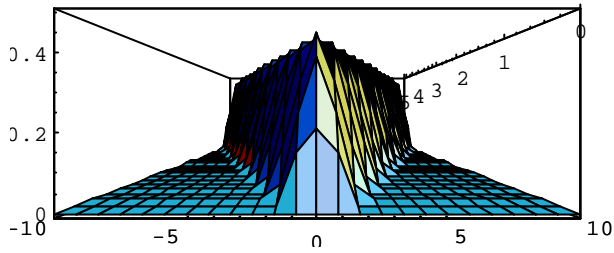
...SurfaceGraphics ...



...SurfaceGraphics ...

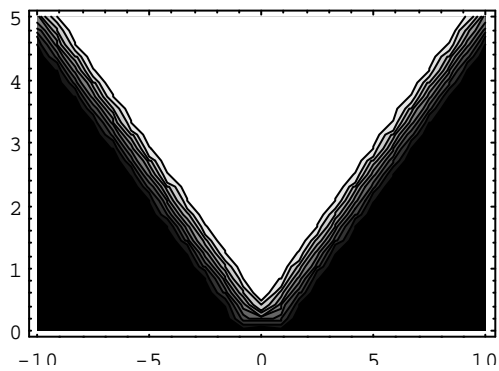


...SurfaceGraphics ...



...SurfaceGraphics ...

Show@ContourGraphics@u2D, AspectRatio@0.7D



...ContourGraphics ...