國立台灣海洋大學九十七學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目： 工程數學

系所名稱： 河工系碩士班（大地工程組、結構工程組、海洋工程組、水資源與環境工程組） ※可使用計算機

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. Vector calculus:

Fig.1

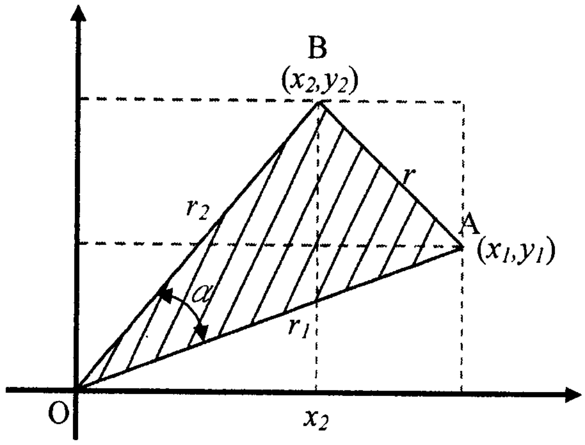
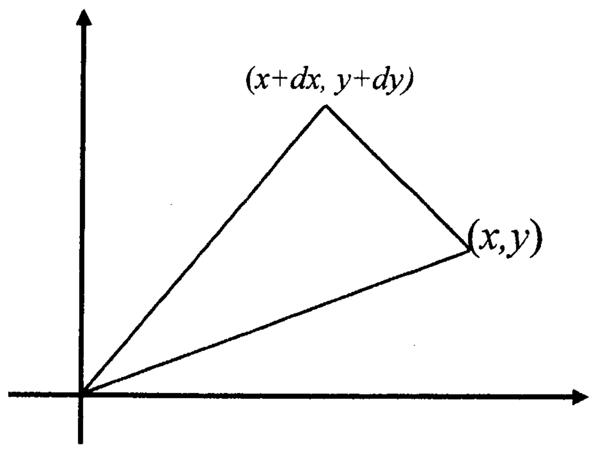


Fig.2



(a). Please determine the area of triangle OAB composed by the three points O:(0,0), A:(2,1), B:(1,2) as shown in Fig.1, where the origin is (0,0), (x1,y1)=(2,1), (x2,y2)=(1,2). (5 %)

(b). Please determine the area of triangle composed by the three points (0,0), (x,y), (x+dx, y+dy) as shown in Fig.2, where (x+dx,y+dy) is the neighborhood of (x,y). (5 %)

(c). What is the Green theorem ? (5 %)

(d). Please determine $\oint_C r\cdot n ds$, where r is the position vector of (x,y) and n is the normal vector of contour C, ds is the path integration, where C is the closed contour along boundaries of triangle OAB: (0,0), (2,1), (1,2) as shown in Fig.1. (5 %)

(e). Please determine the divergence of position vector r i.e., $\nabla\cdot r=?$ where r=x i +y j (5%).

2. Complex variable:

(a). Plot cos(x) versus x and sin(x) versus x. (5 %)

(b) Write down the relationship among e,π and i using $e^{\pi i}=?$.

where $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots,\pi=3.14159\ldots$ and $i^2=-1$. (5 %)

(c). Determine $e^{ix}-\cos(x)-i\ \sin(x)=?$ (5 %)

(d). If the three roots for $z^3=1$ are z_1,z_2,z_3, please find $z_1+z_2+z_3=?$ (5 %)

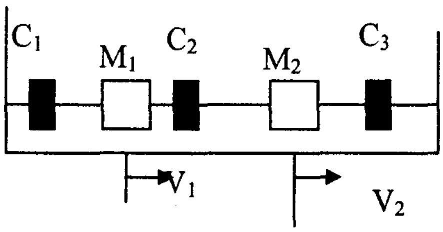
(e). Write down the Cauchy integral formula. (5 %)

3. Fill the blanks in the following table: (10%, 2% for each blank)

Equation:	ODE or PDE	Order of equation	Linear or Nonliner
$\frac{d}{dr}\left[r\frac{du}{dr}\right]=r^2$	ODE	(A)	(B)
$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0$	(C)	2	Linear
$u'''+uu'=0$	ODE	(D)	(E)

4. For a matrix as $\begin{bmatrix} a_1 & a_2 & \cdots & \cdots & a_n \\ a_2 & a_3 & \cdots & \cdots & a_1 \\ \vdots & \vdots & \ddots & & \vdots \\ a_{n-1} & a_n & \cdots & \cdots & a_{n-2} \\ a_n & a_1 & \cdots & \cdots & a_{n-1} \end{bmatrix}$, please prove that there must be an eigenvalue as $a_1+\cdots+a_n$? (10%)

5. Considering a system of dampers with 2 degree of freedoms as shown in the following figure , the damping force is assumed to be proportional to the relative velocity and its direction is in the direction of resisting the relative motion. The inertia force is proportional to the derivative of the velocity vector and the proportional constant is the mass. Assuming M_1 and M_2 are the masses of these two blocks, V_1 and V_2 are the velocities, and C_1, C_2 and C_3 are damping coefficients for the dampers. For simplicity, M_1 and M_2 are assumed to be 1 , C_1, C_2 and C_3 are also 1.
   (A) Please write down the equation of motion for each mass。(5%) (equations should written in terms of velocity)
   (B) Please write down the system of equations as $\mathbf{y}'=\mathbf{A}\mathbf{y}$ according to the result from (A)。(3%)
   (C) Find the eigenvalues and eigenvectors of **A**. (2%)
   (D) What is the general solution for this system of equations? (5%)
   (E) On the phase plane , what kind of critical point one has for this problem? Stable or Unstable? (3%)



6. For an semi-infinite domain $y \le 0$, the steady state of heat equilibrium can be written as a Laplace equation: $\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$ where Φ is the temperature distribution. The boundary condition are given as $\Phi = \Phi_0$ (constant) for $\|x\| \le 1$ and y=0, Φ= 0 for $\|x\| > 1$.and y=0.

Please answer the following questions:

(A) What type of PDE is the Laplace equation? (Elliptical? Hyperbolic? Or parabolic?) (4%)

(B) If one uses symmetry condition on x=0 to analyze the domain for $x \le 0$ and $y \le 0$, what kind of boundary conditions should one give on x=0? (4%)

(C) Following (B), to satisfy the boundary condition on x=0 you can use what transform in x-direction? (Fourier sine transform or Fourier cosine transform?) (4%)