

國立臺灣海洋大學九十九學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目： 工程數學

系所名稱： 河海工程學系碩士班水資源與環境工程組、海洋工程組、結構工程組、大地工程組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. Find the general solution of the following ODE: (10%)

$$y \frac{dy}{dx} + 36x = 0$$

2. Solve the following ODE: (10%)

$$(e^y - ye^x) dx + (xe^y - e^x) dy = 0$$

3. Solve the initial value problem: (10%)

$$y'' + y = 0, \quad y(0) = 3, \quad y'(0) = -0.5$$

4. Solve the nonhomogeneous ODE: (10%)

$$y'' + y = \sec(x) = \frac{1}{\cos(x)}$$

5. Solve the Volterra integral equation of the second kind by Laplace transform: (10%)

$$y(t) - \int_0^t y(\tau) \sin(t-\tau) d\tau = t$$

$$6. \begin{cases} X_1 - 2X_2 - 3X_3 = 6 \\ X_1 + X_2 - X_3 = 5 \\ 3X_1 + 2X_2 = -2 \end{cases} \quad (20\%)$$

- (1) Write the system of equations in matrix form $AX=B$
- (2) Calculate the Eigen-values and Eigen-vectors of matrix A
- (3) Find the Inverse matrix A^{-1}
- (4) Solve $X=A^{-1}B$

$$7. f(t) = \begin{cases} 3\sin(2t) & \text{for } 0 \leq t < 2\pi \\ 0 & \text{for } t < 0 \text{ and for } t \geq 2\pi \end{cases} \quad (30\%)$$

$$H(t-a) = \begin{cases} 1 & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

- (1) Plot the figure of $f(t)$ and $H(t)$
- (2) Expressed $f(t)$ in terms of the Heaviside function $H(t)$
- (3) Find the Taylor series of $f(t)$
- (4) Find the Laplace Transform of $f(t)$
- (5) Find the Fourier series of $f(t)$
- (6) Find the Complex Fourier series of $f(t)$
- (7) Find the Fourier Integral of $f(t)$
- (8) Find the Fourier Transform of $f(t)$
- (9) Find the Fourier Amplitude of $f(t)$
- (10) Find the period of $f(t) = 3\sin(2t)$