

國立海洋大學河海工程學系2004 工程數學研究所考題 (結構與水環)

1. Given the radial vector \mathbf{r} and radial basis function $\phi(r)$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$. Find $\nabla \times \{\phi(r)\mathbf{r}\}$ and $\nabla \cdot \mathbf{r}$. (20 %)

2. Given a matrix K as follows:

$$[K] = \begin{bmatrix} 12 & 6 & -12 & 6 \\ -6 & -4 & 6 & -2 \\ 12 & 6 & -12 & 6 \\ 6 & 2 & -6 & 4 \end{bmatrix}.$$

- (a). Find the rank of $[K]$ matrix. (5 %)
- (b). Find the nullity of $[K]$ matrix. (5 %)
- (c). Find the determinant of $[K]$ matrix. (5 %)
- (d). Solve the two independent solutions of \mathbf{x} which satisfy $[K]\mathbf{x} = \mathbf{0}$. (5 %)

3. If the Laplace transform of $\frac{1}{\sqrt{t}}$ is $P(s)$, find the Laplace transform of \sqrt{t} in terms of $P(s)$. (Hint: two choices: differential operator and multiplication of t) (20 %)

4. Solve the PDE: $u_x - u_y = 0$ subject to $u(0, s) = s^2$. (20 %)

5. Find the integral

$$\int_C \frac{1}{z} dz$$
$$\int_C \frac{1}{z^2} dz$$

where C is an anti-clockwise circle of radius 1 with the origin $(0, 0)$. (10 %)

6. Solve the ODEs (1). $y''(x) = 0$, and (2). $y''(x) + 2y'(x) + y(x) = 0$. (10 %).