Entrance examination of the master program for

Department of Harbor and River Engineering, **Engineering Mathematics**

Time: 8:30~10:10 am Date: April 22, 2000 Subject Code: 2100 –02/2

1. (a) Write out the geometrical meaning of Stokes' theorem. (2%)
   (b) Put down the physical meaning of Stokes' theorem. (2%)
   (c) Show that Green's theorem in the plane is a special case of Stokes' theorem. (2%)
   (d) Verify Stokes' theorem for $\mathbf{F} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ and S the paraboloid : $z = f(x, y) = 1 - (x^2 + y^2),\ z \geq 0$ in Fig. 1. (4%)
2. Solve the equation $y'' + k^2 xy = 0$. (10%)
   (Hint: the substitution of $u = \dfrac{y}{\sqrt{x}}$ and $z = \dfrac{2kx^{\frac{3}{2}}}{3}$ will reduce the given equation into a Bessel equation)
3. (a) Explain the geometrical meaning of the Laplace convolution theorem. (2%)
   (b) What is the physical interpretation of the Laplcae convolution theorem? (2%)
   (c) Use the Laplace convolution theorem to find the inverse of $F(s) = \dfrac{3}{s^2 + 3s - 1}$. (6%)
4. Solve the third order Caucy-Euler equation:
   $x^3 y''' + 9x^2 y'' + 19xy' + 8y = 0$ for $x>0$. (10%)
5. Use matrix method to solve the system of equations:
   $\begin{cases} \dfrac{dy_1}{dt} = -2y_2 \\ \dfrac{dy_2}{dt} = y_1 + 3y_2 \end{cases}$, $t=0$, $y_1=1$, $y_2=3$. (10%)
6. (a) Write down the transformability of Fourier transform, i.e., under what conditions that a function can have its Fourier transform. (3%)
   (b) Please show that if $\hat{f}(w)$ is the Fourier transform of $f(t)$, then $\hat{f}(w-a)$ is the Fourier transform of $e^{iat} f(t)$. (5%)
   (c) Explain why the Fourier transform can obtain the steady-state solution of a differential equa tion but not the transient solution. (2%)
7. For a complex-variable complex-valued function $w(z)=u(x,y)+i\,v(x,y)$ where u and v are real-value functions, $i^2= -1$ and $z=x+i\,y$. Please answer the following problems.
   (a) Write down the Cauchy-Riemann condition and explain the meaning of this condition. (3%)
   (b) Explain the meaning of pole and branch cut. (2%)

(c) Let $w(z) = \dfrac{e^z}{z+1}$, please evaluate the path integrals $\int_C w(z)dz$ as shown in Fig. 2. Notice that there are two paths in the figure, you should evaluate both for full credit, and partial answer will not earn any credit. (5%)

8. (a) Write down three basic types of partial differential equation. (5%)

   (b) Derive that the Laplacian operator in 2-D cylindrical coordinate can be written as

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

   from that $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$. (5%)

   (c) Given a Helmholtz equation $\nabla^2 u = 0$ where k is the wave number and u is the potential and a circular domain with $r \le 1$, the boundary condition is given as $u(1,\theta) = \theta$. Please solve this partial differential equation. (5%)

9. (a) Write down the definition of an orthogonal matrix. (2%)

   (b) Verify if the Householder matrix is an orthogonal matrix. The Householder matrix is defined as $\mathbf{H}_{n\times n} = \mathbf{I}_{n\times n} - 2\mathbf{v}_{n\times 1}\mathbf{v}^T_{1\times n}$ where **H** is the Householder matrix, **I** is an identity matrix and **v** is a unit column vector. (3%)

   (c) For any vector **x**, please verify that $\|\mathbf{Hx}\| = \|\mathbf{x}\|$. (5%)

   (d) Please use 2×2 matrices **H** and **I**, and vectors **v** and **x** in a R^2 space to sketch a diagram to show **x**, **v** and **Hx**. (2%) Please explain the geometrical meaning of the Householder transformation. (3%)

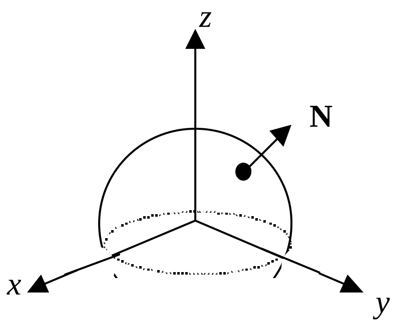

Fig. 1 Surface S.

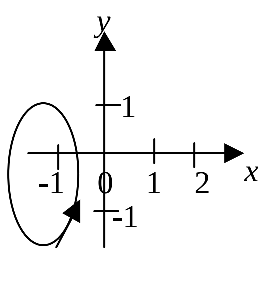

Fig.2a Path 1

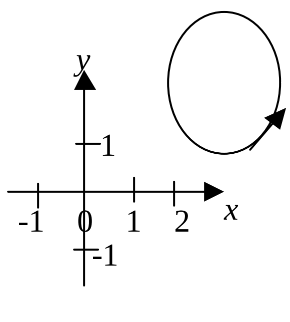

Fig. 2b Path 2