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1. Classify the (a). ordinary differential equation , (b). integral equation, (c). integro-differential equation, (d). logic equation and (e). linear algebraic equation. (20 %)

Equation	Equation type (a, b, c, d, e)
$y''(t) + 4y(t) = 0$	
$x^2 + 3x + 9 = 0$	
$y(t) = \int_0^t y(s)ds$	
$y'(t) = \int_0^t y(s)ds$	
$A \bigcup B = C$	

(註: 請將本表填入 a, b, c, d, e 後, 抄入答案卷才計分)

2. Please explain the Green's theorem (5 %) and the Green's function. (5 %)

3. Given an anti-symmetric matrix W as follows:

$$W = \begin{bmatrix} 0 & \frac{-2}{3} & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{-2}{3} \\ \frac{-1}{3} & \frac{2}{3} & 0 \end{bmatrix}.$$

(a). Calculate $W^T + W =?$ (3 %)
(b). Calculate $W^3 + W =?$ (3 %)
(c). Calculate the determinant for W. (4 %)
where the superscript T denotes transpose.

4. Based on the following relations of the Laplace transform ($\mathcal{L}$),

$$\mathcal{L}\{ty(t)\} = -Y'(s), \ \ where \ \ \mathcal{L}\{y(t)\} = Y(s)$$

the following second order ODE

$$t^2\ddot{y}(t) - 4t\dot{y}(t) + 6y(t) = 0$$

can be transformed to (transform $y(t)$ to $Y(s)$):

$$s^2Y''(s) + asY'(s) + bY(s) = 0$$

where $Y(s)$ is the Laplace transform of $y(t)$, determine a and b. (5 %) If we repeat the Laplace transform with respect to $Y(s)$ again (transform $Y(s)$ to $\bar{Y}(v)$) where $\bar{Y}(v)$ must satisfy

$$v^2\frac{d^2\bar{Y}(v)}{dv^2} + pv\frac{d\bar{Y}(v)}{dv} + q\bar{Y}(v) = 0$$

determine p and q. (5 %)

5. Stokes's theorem (transformation between surface integrals and line integrals)

Let S be a piecewise smooth oriented surface in space and let the boundary S be a piecewise smooth simple closed curve C. Let $\mathbf{F}(x, y, z)$ be a continuous vector function that has continuous first partial derivatives in a domain in space containing S. Then,

$$\int_S (curl\,\mathbf{F}) \cdot \mathbf{n} dA = \oint_C \mathbf{F} \cdot \mathbf{r}' dS$$

where $\mathbf{n}$ is a unit normal vector of S and dependng on $\mathbf{n}$, the integration around C is taken in the sense shown in Fig.1, also $\mathbf{r}' = d\mathbf{r}/ds$ in the unit tangent normal vector and s is the arc length of C.

(a). Write down the physical interpretation of the Stokes's theorem. (3 %)

(b) Verify the Stokes's theorem for $\mathbf{F} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ and S the paraboloid $z = f(x, y) = 1 - (x^2 + y^2), z \geq 0$ in Fig.2. (10 %)

Fig.1 Fig.2

6. By taking the Fourier transform of the equation $\frac{d^2\phi}{dx^2} - K^2\phi = f(x)$, show that its solution $\phi(x)$ can be written as

$$\phi(x) = \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{ikx} \bar{f}(k)}{k^2 + K^2} dk,$$

where $\bar{f}(k)$ is the Fourier transform of $f(x)$. (10 %)

7. Given the one-dimensional heat equation with initial and boundary conditions,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = f(x)$$

$$u_x(0, t) = 0, \text{ for all t}$$

$$u_x(L, t) = 0, \text{ for all t}$$

where c^2 is the thermal diffusivity, L is the bar length, x is space, t is time and $u(x, t)$ is temperature.

(a). Write out the physical meaning of the one-dimensional heat equation. (3 %)

(b). Find a solution of the one-dimensional heat equation using the method of separating variables (or product method). (10 %)

8. Using

$$\int_{-\infty}^{\infty} f(x) dx = 2\pi i \sum Res f(z),$$

show that (14 %)

$$\int_0^{\infty} \frac{dx}{1 + x^4} = \frac{\pi}{2\sqrt{2}}.$$