

92 河工系研究所「工程數學」考試題目：

第一部份：

Find the general solution $y(x)$ of the following equations:

1. $By'' + Dy' + Ey = 0$, where B, D, E are constants (15%)

2. Euler's Equation $x^2 y'' + 5xy' + 4y = 0$, $y(1) = 4$, $y'(1) = 4$ (15%)

3. Write the following function values in the form of $a + bi$ (10%)

(a). e^{2+5i} (b). $\text{Cos}(3+3i)$

4. Using Cauchy's Theorem to evaluate the complex integration: $\oint_C \frac{2z+1}{z^2+3iz} dz = ?$

Where C is the circle $|z+3i|=2$ (i.e. $z = x + iy$ and $x^2+(y+3)^2 = 4$) of radius 2 and center $(0, -3i)$. (10%)

第二部份：

1. Given the three matrices (15 %)

$$Q_1 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \quad Q_2 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \quad Q_3 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

and a position vector \tilde{y} , write down the geometric relationship of

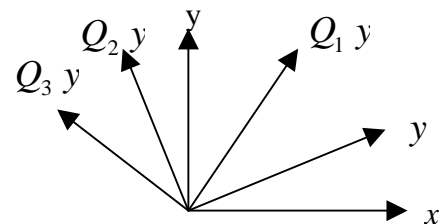
(a) $Q_1 \tilde{y}$ and \tilde{y} .

(b) $Q_3 \tilde{y}$ and \tilde{y} .

(c) Can you find a matrix Q such that $Q^T Q = I$ (orthogonal) and $Q = Q^T$?

(d) $Q_3 Q_3 = ?$

(e) $\det Q_1 = ?$



2. Given $A\tilde{x} = \tilde{b}$,

$$\text{where } A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 7 & 6 & 19 \end{bmatrix} \text{ and } \tilde{b} = \begin{bmatrix} 8 \\ 8 \\ 32 \end{bmatrix}.$$

(a) Find the rank of $[A]$. (2 %)

(b) Find the nontrivial solution $\{\phi\}$ such that

$$[A]^T \tilde{\phi} = 0,$$

where T denotes the transpose. (2 %)

(c) Determine $\{\tilde{b}\}^T \{\phi\} = ?$ (2 %)

(c) Please write down the Fredholm Alternative Theorem using this example. (4%)

3. (a) Given the equation $x^2 - xy + y^2 = 1$, is the shape ellipse, hyperbolic or parabolic curve ? (3 %)

(b) Transform $x^2 - xy + y^2 = 1$ to quadratic form $\begin{Bmatrix} x \\ y \end{Bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = 1$.

Find the symmetric matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ (3 %)

(c) Find the eigenvalues (λ_1, λ_2) and eigenvectors (v_1, v_2) of A . (3%)

(d) Transform $x^2 - xy + y^2 = 1$ to $\lambda_1 \bar{x}^2 + \lambda_2 \bar{y}^2 = 1$ by $\begin{Bmatrix} x \\ y \end{Bmatrix} = Q \begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix}$.

Find Q and determine $Q^T A Q$? (3%)

(e) Plot the figure in terms of x - y system and new system. (3%)

4. Explain the mathematical definition of Green's function and its physical meaning. (10%)