國立臺灣海洋大學河海工程研究所 1995 工程數學入學考

1(a). Using Green's theorem, derive the integral formula (8%)

$$\int_C \frac{\partial u}{\partial n} dS = \int_D \int \nabla^2 u(x,y) dxdy$$

for a twice continuously differential function u, where S denotes arc length on C, $\partial u/\partial n$ refers to the outward normal derivative of u on C and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

1(b). Calculate the line integral (5%)

$$\int_C (x+y^2)dx - xydy$$

where C is the sides of the square with vertices at (1,0), (0,1),(-1,0) and (0,-1) oriented in the counterclockwise direction.

2(a). Explain the geometrical and physical meaning of convolution integral.(6%)

2(b). Solve the initial value problem (7%)

$$y'' + 2y = r(t), y(0) = y'(0) = 0$$

where

$$r(t) = \begin{cases} 1.0 & if \;\; 0 < t < 1 \\ 0. & otherwise \end{cases}$$

3. Evaluate (10%)

$$\int_1^x \int_1^t \frac{ln(z)}{z^2} dzdt$$

4. Using the generating function for the Legendre Polynomials $P_n(x)$, show that

(a). $(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0, n = 1,2,3...(7\%)$

(b). $nP_n(x) - xP_n'(x) + P_{n-1}'(x) = 0, n = 1,2,3...(7\%)$

5. If $f(t)$ is defined by $\dot{f}(t) = \delta(t)$ or

$$f(t) = \begin{cases} 0.5, & for \;\; t > 0, \\ -0.5, & for \;\; t < 0, \end{cases}$$

(a). Find the Fourier transform of $f(t)$ and explain why no real part exists.(5%)
(b). Using (a), find (5%)

$$\int_{-\infty}^{\infty} \frac{sin(\omega)}{\omega} d\omega = ?$$

(c). Using Parseval's theorem, find (5%)

$$\int_{-\infty}^{\infty} \mid \frac{1}{a+b\omega i} \mid^2 d\omega = ?, where \;\; a,b \;\; are \;\; constants, \;\; i^2 = -1$$

(d). Using residue theorem, find (5%)

$$\int_{-\infty}^{\infty} \mid \frac{1}{a+b\omega i} \mid^2 d\omega = ?$$

(e). If the results of (c) and (d) are the same, do you find any physical meaning ? (5%)

6.(a). Explain characteristic line (3 %) and diamond rule. (2 %)
(b). How many methods can be applied to solve the following PDE ? (5%)

$$u_{tt} = u_{xx}, \quad for \;\; 0 < x < 1, \;\; t > 0$$

with initial conditions

$$u(x,0) = 0, u_t(x,0) = 0$$

and boundary conditions

$$u(0,t) = a(t), u(1,t) = b(t), where \;\; a(t), b(t) \;\; are \;\; known$$

(c). Please give comments on their advantages and disadvantages.(5%)
(d). Go through by one of the methods you can.(10 %)