國立臺灣海洋大學河海工程研究所 1997 工程數學入學考

1. The five ODEs are defined for $y(x)$ as follows:

$$
\begin{aligned}
y' &= y^{3/5}, & (1)\\
y' &= (y^2-4)(y-4), & (2)\\
y' &= \frac{2xy}{1+y^2}, & (3)\\
y' &= 1+x+y, & (4)\\
y &= xy' + y'y. & (5)
\end{aligned}
$$

Please fill in the Table 1 for its nonlinearity and order. (請將表 1 抄入答案紙, 才予以計分, 填入 YES 或 NO 與階次) (10 %)

Table 1: Solutions mapping to each ODE

ODE	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5
linear ?					
order ?					

2. Given $U(x,y) = ln\sqrt{(x^2+y^2)}$, and polar coordinates $x = r\,cos(\theta),\ y = r\,sin(\theta),\ dA = rdrd\theta$.
(1). Find $\mathbf{F}$ where $\mathbf{F} = \nabla U$.(5%)
(2). Find $\int \mathbf{F}\cdot\mathbf{n}\ ds$ along the boundary contour ds of unit circle. (5%) (Note that $\mathbf{n}$ is the normal vector on the boundary contour of unit circle).
(3). Find $\int\int \nabla\cdot\mathbf{F}\ dA$ on the area of unit circle. (5%)
(4). Divergence theorem tells us that $\int\int \nabla\cdot\mathbf{F}\ dA = \int \mathbf{F}\cdot\mathbf{n}\ ds$. Are the results of (2) and (3) the same ? Why ! (5%)

3. The system of ODE equations

$$\dot{x}(t) = x - 2y,$$

$$\dot{y}(t) = 3x - 4y$$

can be formulated as

$$\dot{\mathbf{x}} = A\mathbf{x},$$

where

$$\mathbf{x} = \{x \quad y\}^T,\ \ A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}.$$

(1). Find the eigenvalues and eigenvectors for A. (4 %)
(2). If the initial condition satisfies $x(0) = y(0)$, what is the relation between y and x ? (2 %)
(3). If the initial condition satisfies $3x(0) = 2y(0)$, what is the relation between y and x ? (2 %)
(4). What is the geometric meaning of eigenvector for this problem in the phase plane, *i. e.*, the figure of $y(t)$ versus $x(t)$? (2 %)

4.(1). Write down the definition of Fourier transform and inverse Fourier transform for $f(t)$. (4 %)
4.(2). $\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\ e^{-i\omega t}\ dt\ e^{i\omega t}\ d\omega = ?$ (3 %), $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\ e^{-i\omega t}\ dt\ e^{-i\omega t}\ d\omega = ?$ (3 %)

5. Given the sine integral $\int_0^\infty \frac{sin(x)}{x} dx = \frac{\pi}{2}$.
(1). Show that the sine integral by using the method of real improper integral, that is integrand having a pole on the real axis. (5 %)
(2). Show that the sine integral by using the method of Cauchy integral theorem. (7 %)
(3). Show that the sine integral by using the method of Fourier transform. (8 %)
(4). Show that the sine integral by using the method of Laplace transform. (5 %)

6.(1). Under what condition the power series can be represented as Taylor's series ? (3 %)
6.(2). Derive Taylor's series from the power series. (5 %)
6.(3). Explain the method of Frobenius. (3 %)
6.(4). Write out that the difference and relationship among power series, Taylor's series and Mclaurin's series series, Frobenius's series and Laurent's series. (8 %)
6.(5). Does the Laurent series $g(z)$ at $z = 0$ in a Taylor series where $g(z) = \frac{sin(z)}{z^3 - z}$. (3 %)
6.(6). Compute the residue at all singularity of $g(z)$. (3 %)