國立臺灣海洋大學河海工程研究所 1998 工程數學入學考

1. Solve the ordinary differential equation

$$y'' + 4y' + 3y = 65\, cos(2x)$$

by using

(a). the method of undetermined coefficient. (5%)
(b). the method of variation of parameter. (5%)
(c). complex method. (2%)
(d). the Heaviside inverse operator method. (3%)

2. Given a simply supported beam applied by a concentrated load as shown in Figure 1. Find the elastic surve by Laplace transform method. (10%)

3. Derive the integral formula

$$J_0(x) = \frac{2}{\pi} \int_0^{\pi/2} cos[x sin(\theta)] d\theta.$$

Hint:

$$e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}$$

$$J_\mu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+\mu}\, n!\, \Gamma(n+\mu+1)} x^{2n+\mu}$$

where $J_\mu(x)$ is the Bessel function of the first kind of order μ. (10%)

4. Given

$$A = \begin{bmatrix} -1 & 1 & -3 \\ 1 & 0 & 2 \\ 2 & -1 & 4 \end{bmatrix},$$

(a). Find eigenvalues and eigenvectors. (3 %)
(b). Find P such that $P^{-1}AP = J$, where J is Jordan Canonical form. (5 %)
(c). Write out the physical significance of the Jordan canonical form J matrix. (2%)
(d). Find $f(A) = A^{10} = ?$ (5 %)

5. Given $U(x, y, z) = \frac{1}{\sqrt{(x^2+y^2+z^2)}}$,

(a). Determine the gradient of U, i.e., $\mathbf{F} = \nabla U$ in terms of xyz coordinates. (3%)
(b). Determine the curl of $\mathbf{F}$, i.e., $\nabla \times \mathbf{F}$, in terms of xyz coordinates. (3%)
(c). Determine the Laplacian of $U(x, y, z)$, i.e., $\nabla^2 U(x, y, z) =?$ (3%)

6. Given $f(t)$ is a real function, the Fourier tansform of $f(t)$ is

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

where $\mathcal{F}$ is operator of Fourier tansform. The inverse Fourier transform is defined as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

Please fill in YES or NO in the following Table. (9%)

	$F^*(\omega) = F(-\omega)$	$\mathcal{F}\{\mathcal{F}\{f(t)\}\} = 2\pi f(-t)$	$\mathcal{F}\{f(t-a)\} = e^{-i\omega a}F(\omega)$
Y or N			

where $*$ is complex conjugate and $i^2 = -1$.

7. According to the Figure 2, determine the following contour integrals in complex palne. (20 %)

$$\begin{aligned}
A &= C.P.V. \int_{C_1} \frac{1}{(z-0.5)} dz \\
B &= \int_{C_1+C_2+C_3+C_4} \frac{1}{(z-0.5-0.5i)} dz \\
C &= \int_{C_1+C_2+C_3+C_4} \frac{1}{(z-0.5-0.5i)^2} dz \\
D &= \int_{C_2+C_3+C_4} \frac{1}{(z-0.5)} dz
\end{aligned}$$

where $z = x + yi, i^2 = -1$, $C.P.V.$ is the Cauchy principal value, C_1 is the line element from (0,0) to (1,0), C_2 is the line element from (1,0) to (1,1), C_3 is the line element from (1,1) to (0,1), C_4 is the line element from (0,1) to (0,0). and a is the point of (0.5, 0) on C_1.

Fig.2

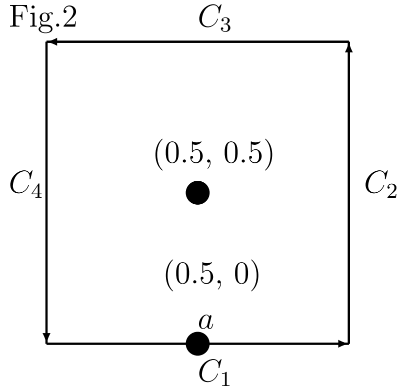
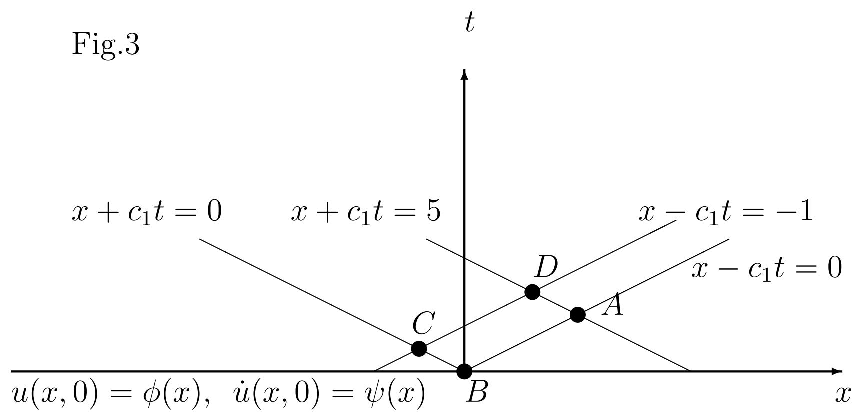


Fig.3



8. Given the D'Alembert solution as shown in Figure 3:

$$u(x,t) = \frac{1}{2}\phi(x + c_1t) + \frac{1}{2}\phi(x - c_1t) + \frac{1}{2c_1}\int_{x-c_1t}^{x+c_1t} \psi(x)dx$$

which satisfies the governing equation

$$u_{tt} = c_1^2 u_{xx}, \quad -\infty < x < \infty, \quad t > 0$$

and the Cauchy data

$$u(x,0) = \phi(x), \quad \dot{u}(x,0) = \psi(x),$$

where c_1 is wave velocity. What is characteristic curve (3 %) ? What is influence area(zone) (3 %) ? What is domain of dependence (3 %) ? Is $u_A + u_C = u_B + u_D$ right (3%) ? Note that u_A, u_B, u_c and u_D are the values of $u(x,y)$ at A, B, C and D points in Figure 3.