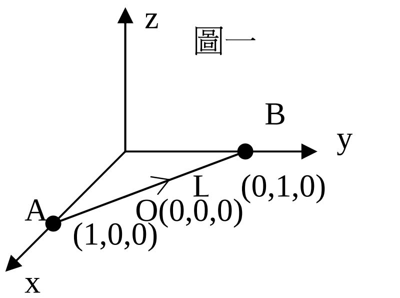
年級：_____________　　姓名：____________________　　學號：________________

國立台灣海洋大學河海工程學系２００４工程數學解答 (March 29. 2005) 結構大地海工與水環

１. (1) $\nabla\cdot\underset{\sim}{r}=?$ where $\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$.(2%)



ANS $\nabla\cdot\underset{\sim}{r}=3$

(2) Line integral $\oint_C \underset{\sim}{r}\cdot\underset{\sim}{n}\,ds=?$

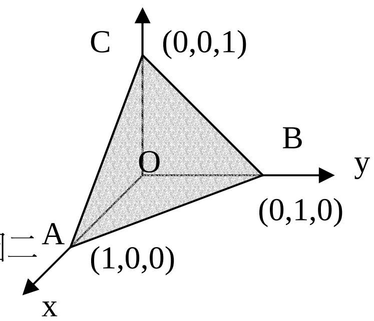
where C is the closed loop of OAB.(圖一) (4%)

ANS $x+y=1,\quad \oint_C \underset{\sim}{r}\cdot\underset{\sim}{n}\,ds=\oint_c (x\underset{\sim}{i}+y\underset{\sim}{j})\cdot(dy\,\underset{\sim}{i}-dx\,\underset{\sim}{j})=\int_0^1 dy=1$

(3) Surface integral: $\iint_S \underset{\sim}{r}\cdot\underset{\sim}{n}\,dS=?$

where S is the surface of plane ABC.(圖二) (4%)

(Note that $\underset{\sim}{n}$ is the normal vectors of ds and dS, respectively)



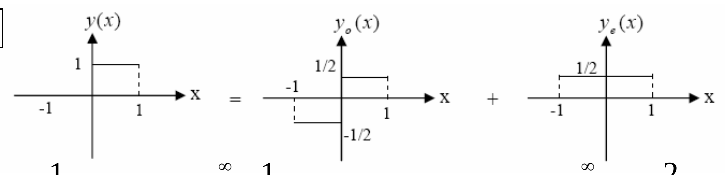
圖二

ANS $\iint_{s_2} r\cdot n ds=\iiint\nabla\cdot r dV=3V=\frac{1}{2}$

2. Give a function $y(x)$ with a period 2 and
$y(x)=0,-1<x<0$ and $y(x)=1,0<x<1$

(1) Decompose the function into even function of $y_e(x)$ and odd function of $y_o(x)$ (2%)
(2) Plot $y(x)$, $y_e(x)$ and $y_o(x)$. (3%)
(3) Expand $y_e(x)$ and $y_o(x)$ into Fourier series. (5%)
(4) Is termwise (term by term) differentiation legal with respect to any Fourier series ? (5%)

ANS



$$y_e(x)=\frac{1}{2};\quad y_o(x)=\sum_{n=1}^{\infty}\frac{1}{n\pi}(1-(-1)^n)\sin(n\pi x)=\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin((2k+1)\pi x);$$

$$y(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1}{n\pi}(1-(-1)^n)\sin(n\pi x)=\frac{1}{2}+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin((2k+1)\pi x);$$

$$a_0=\frac{1}{4}[\int_{-2}^{-1}dx+\int_0^1 dx]=\frac{1}{2},a_n=\frac{1}{2}[\int_{-2}^{-1}\cos(\frac{n\pi}{2}x)dx+\int_0^1\cos(\frac{n\pi}{2}x)dx]=0$$

$$b_n=\frac{1}{2}[\int_{-2}^{-1}\sin(\frac{n\pi}{2}x)dx+\int_0^1\cos(\frac{n\pi}{2}x)dx]=\frac{1}{n\pi}[\cos(n\pi)+1-2\cos(\frac{n\pi}{2})]$$

$$y(x)=\frac{1}{2}+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin((2k+1)\pi x)$$

T=4 與 T=2 做傅立業展開其結果相同。

3. Complex variable

(1) $\oint_C \frac{1}{z} dz = ?$ where C is the unit circle in a counterclockwise direction. (2%)

ANS $\oint_C \frac{1}{z} dz = 2\pi i$

(2) What is the definition of Cauchy principal value (CPV) ? (3%)

ANS $CPV\int_{-\infty}^{\infty} \frac{f(x)}{x} dx = \lim_{\varepsilon\to 0} \int_{-\infty}^{\varepsilon} + \int_{\varepsilon}^{\infty} \frac{f(x)}{x} dx$

(3). $CPV\int_{-\infty}^{\infty} \frac{\cos(mx)}{x-a} dx = ?,$ for a real, $m > 0$ (4%)

ANS $CPV\int_{-\infty}^{\infty} \frac{\cos(mx)}{(x-a)} dx = -\pi \sin(ma)$

(4). $CPV\int_{-\infty}^{\infty} \frac{\sin(mx)}{x-a} dx = ?,$ for a real, $m > 0$ (4%)

ANS $CPV\int_{-\infty}^{\infty} \frac{\sin(mx)}{(x-a)} dx = \pi \cos(ma)$

(5). What is Hilbert transform ? (2%)

ANS Hilbert transform $f(t) \to CPV\int_{-\infty}^{\infty} \frac{f(\tau)}{\pi(t-\tau)} d\tau = \mathrm{H}(f(t))$

4. Solve the following partial differential equation.

$yu_x - xu_y = 3x$ subject to $u(x,0) = x^2$ Solve $u(x,y) = ?$ (10%)

ANS

Method I

$$\begin{cases} yu_x - xu_y = 3x \\ \frac{dx}{dt}\frac{\partial u}{\partial x} - \frac{dy}{dt}\frac{\partial u}{\partial y} = \frac{\partial u}{\partial t} \end{cases} \Rightarrow \begin{cases} \frac{dx}{dt} = y \cdots (1) \\ \frac{dy}{dt} = -x \cdots (2) \\ \frac{du}{dt} = 3x \cdots (3) \end{cases}$$

$\frac{(2)}{(1)} \Rightarrow (x^2 + y^2) = s^2$ $\frac{(3)}{(2)} \Rightarrow u = -3y + s^2 = -3y + (x^2 + y^2)$

Method II

£ ① ⇒ ŠX-s =Y £ ② ⇒ ŠY=-X

£ ③ ⇒ ŠU-s^2 =3 X

by 克萊瑪 rule or 聯立方程

$X = s\text{Š}/(\text{Š}^2+1) \Rightarrow \text{£}^{-1}\, X = x(s,t) = s\cos t$

$Y = -s/(\text{Š}^2+1) \Rightarrow \text{£}^{-1}\, Y = y(s,t) = -s\sin t$

$U = -3s/(\text{Š}^2+1) + s^2/\text{Š} \Rightarrow \text{£}^{-1} U = u(s,t) = -3s\sin t + s^2$

So, we know $\Rightarrow u(x,y) = -3y + (x^2 + y^2)$