國立台灣海洋大學九十三學年度研究所碩士班招生考試試題

系所名稱：河海工程學系碩士班(海工組)　　*答案以橫式由左至右書寫於答案卷上！

科目名稱：工程數學　　*使用計算機

01. Find all of the singular points and classify each singular point as regular or irregular. $(x^3-2x^2-7x-4)y''-2(x^2+1)y'+(5x^2-2x)y=0$ (10%)

02. Show the determinant that

$$\begin{vmatrix} 1+\alpha^2+\alpha^4 & 1+\alpha\beta+\alpha^2\beta^2 & 1+\gamma\alpha+\gamma^2\alpha^2 \\ 1+\alpha\beta+\alpha^2\beta^2 & 1+\beta^2+\beta^4 & 1+\gamma\beta+\gamma^2\beta^2 \\ 1+\gamma\alpha+\gamma^2\alpha^2 & 1+\gamma\beta+\gamma^2\beta^2 & 1+\gamma^2+\gamma^4 \end{vmatrix} = (\gamma-\alpha)^2(\gamma-\beta)^2(\alpha-\beta)^2 \quad (10\%)$$

03. Evaluate the determinant.

$$\begin{vmatrix} 16 & 14 & 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 & 0 \\ 1 & 3 & 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 0 & 4 & -9 \end{vmatrix} \quad (10\%)$$

04. Let **A** be a real symmetric matrix. Prove that **A** is positive definite. (10%)

05. Find the streamlines of the vector field, and find the particular streamline through the given point. $\mathbf{F}=(1/x)\mathbf{i}+e^x\mathbf{j}-\mathbf{k}$; (2, 0, 4) (10%)

06. Find the equations of the tangent plane and normal line to the surface at the point. $2x-4y^2+z^3=0$; $(-4,\ 0,\ 2)$ (10%)

07. Let $u(x, y)$ be continuous with continuous first and second partial derivatives on a simple, closed path C and throughout the interior D of C. Show that

$$\oint_C -\frac{\partial u}{\partial y}dx+\frac{\partial u}{\partial x}dy=\iint_D\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]dA \quad (10\%)$$

08. Evaluate the integral. $\int_{-\infty}^{\infty}\frac{x^2\cos(x)}{1+x^6}dx$ (10%)

09. Apply separation of variables to solve the following mixed boundary value problem. Find the general solution of $\phi(x,z)$. (20%)

$$\frac{\partial^2\phi(x,z)}{\partial x^2}+\frac{\partial^2\phi(x,z)}{\partial z^2}=0 \quad \text{for } -\infty<x<\infty,\ -h\le z\le 0$$

$$\frac{\partial\phi(x,z)}{\partial z}=\frac{\sigma^2}{g}\phi(x,z),\quad z=0$$

$$\frac{\partial\phi(x,z)}{\partial z}=0,\quad z=-h$$