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Trefftz method: an overview

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The aim of this paper is to review the existing formulations of 'Trefftz method'. The Trefftz formulations are classified into the direct and the indirect formulations and then, compared with other boundary-type solution procedures, such as boundary element, singularity, charge simulation and surface charge methods, in order to establish the identity of the method.

Key words: Trefftz method, direct formulation, indirect formulation, boundary element method, singularity method.

1 INTRODUCTION

The method known as 'Trefftz method' was firstly presented by Trefftz[1] in 1926 and since then, many researchers have studied it. The Trefftz method can be classified into the category of the boundary-type solution procedures. The problem can be solved by the boundary discretization alone when the object is governed by the linear and homogeneous differential equation. Therefore, input data generation is much easier than the domain-type solution procedures. Moreover, the Trefftz formulation is regular and thus, easier than the boundary element method of singular property. Although the Trefftz method has the above-mentioned various advantages, it is less popular than the other solution procedures such as finite difference, finite element and boundary element methods. The aim of this paper is to overview the existing Trefftz formulations and to compare them with the other boundary-type solution procedures in order to establish the identity of the 'Trefftz method'.

The Trefftz formulations can be classified into the indirect and the direct ones. In the indirect formulation,[1–87] which is thought to be the original one presented by Trefftz, the solution of the problem is approximated by the superposition of the functions satisfying the governing equation and then, the unknown parameters are determined so that the approximate solution satisfies the boundary condition by means of the collocation, the least square or the Galerkin method. In the direct formulation, which is relatively new one presented by Cheung and his co-workers,[3–6] the weighted residual expression of the governing equation is derived by taking the regular function satisfying the governing equation as the weighting function and then, the boundary integral equation is obtained by applying twice the Gauss–Green formula to it. The integral equation, like the boundary element method, is discretized and solved for the boundary unknowns. In this paper, these formulations will be explained briefly in order to compare with the other boundary-type solution procedures. Finally, we should recognize that the indirect and the direct formulations of the Trefftz method are closely related to the direct and indirect ones of the ordinary boundary element method. By the way, in this paper, the terminology 'boundary element method' denotes the conventional boundary element method (or boundary integral equation method) alone and the terminology 'boundary-type solution procedure' is used as a general term including the boundary element, the singularity, the charge simulation, the surface charge and the Trefftz methods.

In Sections 2 and 3, the indirect and the direct Trefftz formulations will be explained briefly. In Section 4, the Trefftz method is compared with the other boundary-type solution procedures. Section 5 is the conclusion and then, the similarities and the differences between the boundary element method and the Trefftz method will be discussed once more.