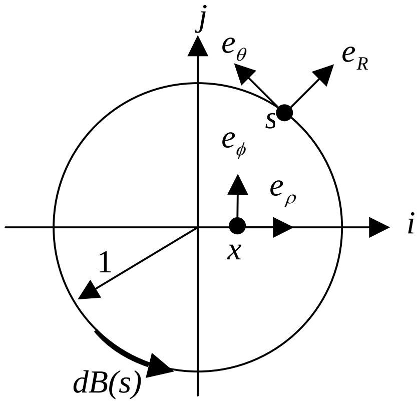
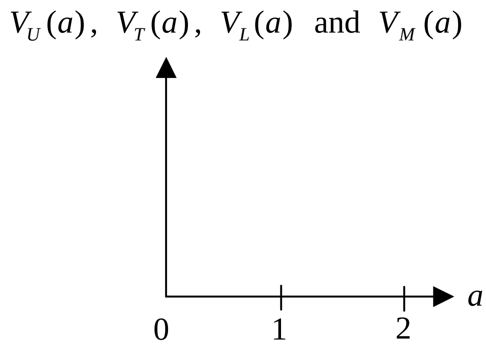
BEM 期末考 (Open Book) Jan. 19, 14:00~16:00, 2005

1. Given $U(s,x)=\ln r$, $T(s,x)=\dfrac{\partial U(s,x)}{\partial n_s}$, $L_\rho(s,x)=\dfrac{\partial U(s,x)}{\partial \rho}$, $M_\rho(s,x)=\dfrac{\partial T(s,x)}{\partial \rho}$,

$L_\phi(s,x)=\dfrac{1}{\rho}\dfrac{\partial U(s,x)}{\partial \phi}$ and $M_\phi(s,x)=\dfrac{1}{\rho}\dfrac{\partial T(s,x)}{\partial \phi}$





$x=(\rho,\phi)$, $s=(R,\theta)$

Determine

(1) Plot $V_U(a)$ versus a where $V_U(a)=\int_B U(s,x)1dB(s)$, $x=(a,0)$, 0< a <2.

(2) Plot $V_T(a)$ versus a where $V_T(a)=\int_B T(s,x)1dB(s)$, $x=(a,0)$, 0< a <2.

(3) Plot $V_\rho(a)$ versus a where $V_{L_\rho}(a)=\int_B L_\rho(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\rho$,

0< a <2.

(4) Plot $V_\rho(a)$ versus a where $V_{M_\rho}(a)=\int_B M_\rho(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\rho$,

0< a <2.

(5) Plot $V_\phi(a)$ versus a where $V_{L_\phi}(a)=\int_B L_\phi(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\phi$,

0< a <2.

(6) Plot $V_\phi(a)$ versus a where $V_{M_\phi}(a)=\int_B M_\phi(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\phi$,

0< a <2.

(7) Discuss the continuity when the six functions go through 1 ($1^- < x < 1^+$). (35%)

2. Given $[A]=\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, find α, β, γ such that $[A]=\alpha C^3+\beta C+\gamma C^2$.

Determine the three eigenvalues of [A] where $C=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. (30%)

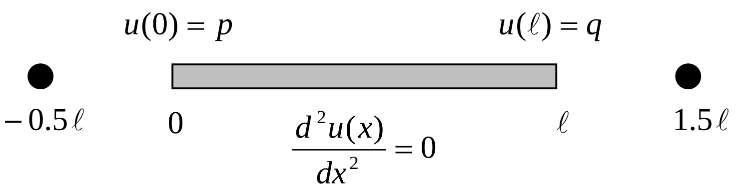
3. Solve the rod problem ($0<x<\ell$) using the direct *UT* method with the fundamental solution $U(s,x)=\frac{1}{2}|s-x|+C$ for the Dirichlet problem $u(0)=p$ and $u(\ell)=q$.

(1) Find [*U*] and [*T*] such that $[U]\begin{Bmatrix} t(0) \\ t(\ell) \end{Bmatrix}=[T]\begin{Bmatrix} u(0) \\ u(\ell) \end{Bmatrix}$.

(2) For *C*=0, is there any degenerate scale ?

(3) For $C=-\frac{1}{4}$, determine the degenerate scale of ℓ such that $\det[U]=0$.

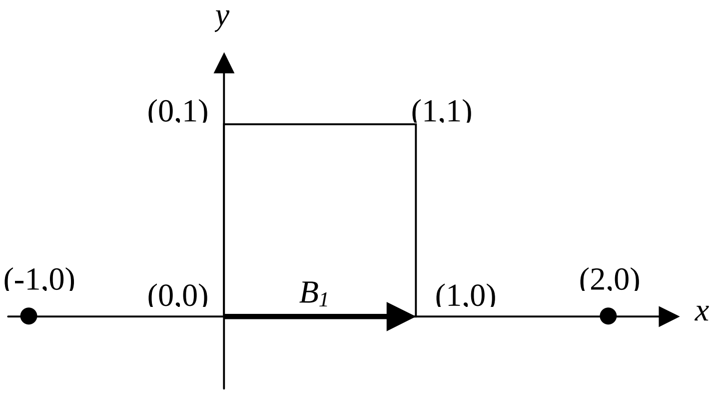
(4) By using the indirect BEM, $u(x)=U(x,s_1)\phi_1+U(x,s_2)\phi_2$ where $s_1=-0.5\ell$ and $s_2=1.5\ell$. Please find *C* such that $\det[U]=0$ where $[U]\begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}=\begin{Bmatrix} p \\ q \end{Bmatrix}$ (20%)



4. Determine

(a) $\int_{B_1} U(s,x)dB(s)=?$ x=(-1,0)

(b) $\int_{B_1} T(s,x)dB(s)=?$ (10%)



5. Please write the five advantages and five disadvantages of BEM. (10%)

6. What is the rigid body test (constant potential) to determine the diagonal terms of [*T*] and [*M*] matrices ? How to apply the technique to degenerate boundary ? (10%)