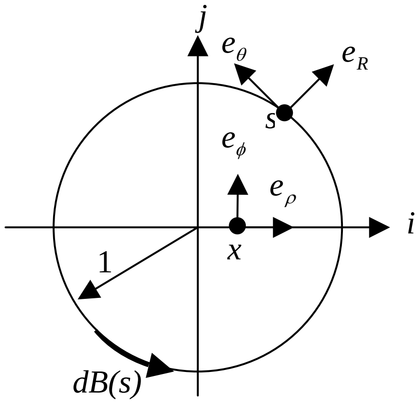
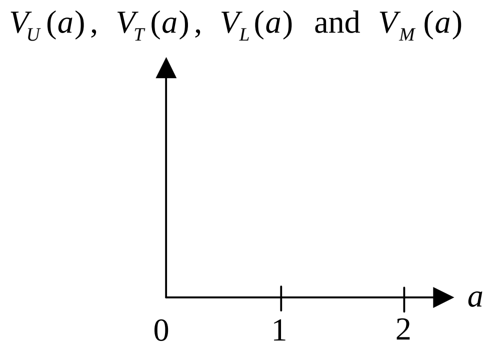
BEM 期末考 (Open Book) Jan. 19, 14:00~16:00, 2005

1. Given $U(s,x)=\ln r$, $T(s,x)=\dfrac{\partial U(s,x)}{\partial n_s}$, $L_\rho(s,x)=\dfrac{\partial U(s,x)}{\partial \rho}$, $M_\rho(s,x)=\dfrac{\partial T(s,x)}{\partial \rho}$,

$L_\phi(s,x)=\dfrac{1}{\rho}\dfrac{\partial U(s,x)}{\partial \phi}$ and $M_\phi(s,x)=\dfrac{1}{\rho}\dfrac{\partial T(s,x)}{\partial \phi}$





$x=(\rho,\phi)$, $s=(R,\theta)$

Determine

(1) Plot $V_U(a)$ versus a where $V_U(a)=\int_B U(s,x)1dB(s)$, $x=(a,0)$, 0< a <2.

(2) Plot $V_T(a)$ versus a where $V_T(a)=\int_B T(s,x)1dB(s)$, $x=(a,0)$, 0< a <2.

(3) Plot $V_\rho(a)$ versus a where $V_{L_\rho}(a)=\int_B L_\rho(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\rho$,

0< a <2.

(4) Plot $V_\rho(a)$ versus a where $V_{M_\rho}(a)=\int_B M_\rho(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\rho$,

0< a <2.

(5) Plot $V_\phi(a)$ versus a where $V_{L_\phi}(a)=\int_B L_\phi(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\phi$,

0< a <2.

(6) Plot $V_\phi(a)$ versus a where $V_{M_\phi}(a)=\int_B M_\phi(s,x)1dB(s)$, $x=(a,0)$ and $n_x=e_\phi$,

0< a <2.

(7) Discuss the continuity when the six functions go through 1 ($1^- < x < 1^+$). (35%)

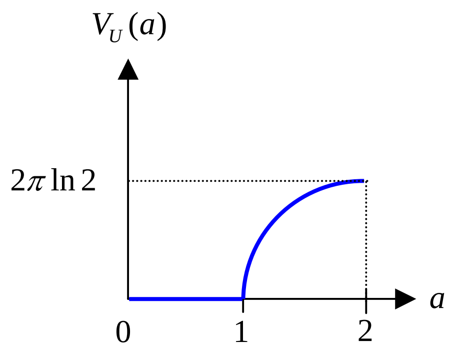
Ans:

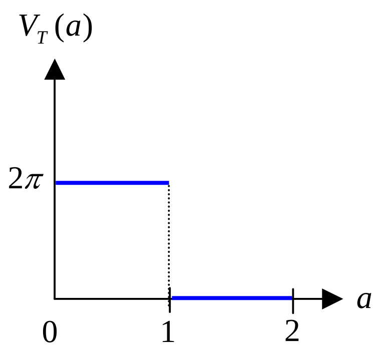
(1) $V_U(a) = 0,\ 0 < a < 1$

$V_U(a) = 2\pi \ln a,\ 1 < a < 2$

(2) $V_T(a) = 2\pi,\ 0 < a < 1$

$V_T(a) = 0,\ 1 < a < 2$



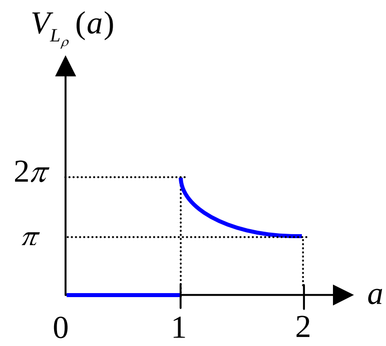


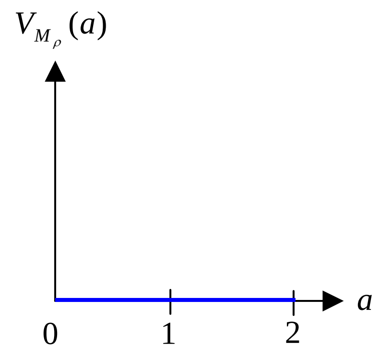
(3) $V_{L_\rho}(a) = 0,\ 0 < a < 1$

$V_{L_\rho}(a) = \dfrac{2\pi}{a},\ 1 < a < 2$

(4) $V_{M_\rho}(a) = 0,\ 0 < a < 1$

$V_{M_\rho}(a) = 0,\ 1 < a < 2$



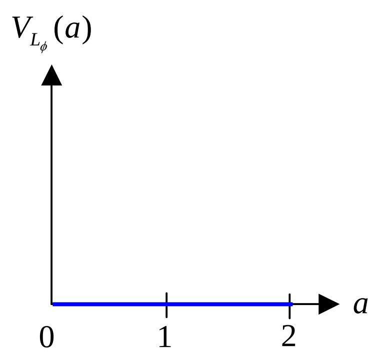


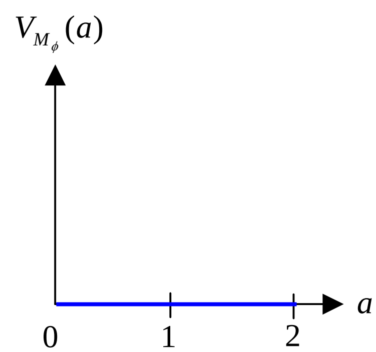
(5) $V_{L_\phi}(a) = 0,\ 0 < a < 1$

$V_{L_\phi}(a) = 0,\ 1 < a < 2$

(6) $V_{M_\phi}(a) = 0,\ 0 < a < 1$

$V_{M_\phi}(a) = 0,\ 1 < a < 2$





(7) U, M_ρ 連續；T, L_ρ 不連續；L_ϕ 連續；M_ϕ 不連續(此時看不出)

2. Given $[A] = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, find α, β, γ such that $[A] = \alpha C^3 + \beta C + \gamma C^2$.

Determine the three eigenvalues of [A] where $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. (30%)

Ans: $\alpha = 0, \beta = -1, \gamma = 1$, $\lambda = 0, -\sqrt{3}i, \sqrt{3}i$

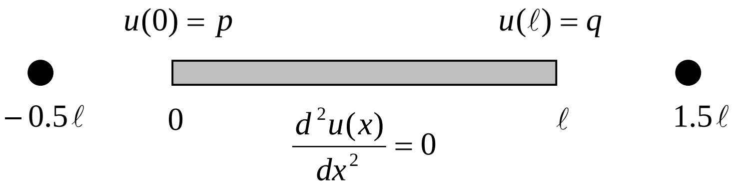
3. Solve the rod problem ($0 < x < \ell$) using the direct *UT* method with the fundamental solution $U(s,x) = \frac{1}{2}|s-x| + C$ for the Dirichlet problem $u(0) = p$ and $u(\ell) = q$.

(1) Find [*U*] and [*T*] such that $[U]\begin{Bmatrix} t(0) \\ t(\ell) \end{Bmatrix} = [T]\begin{Bmatrix} u(0) \\ u(\ell) \end{Bmatrix}$.

(2) For *C*=0, is there any degenerate scale ?

(3) For $C = -\frac{1}{4}$, determine the degenerate scale of ℓ such that $\det[U] = 0$.

(4) By using the indirect BEM, $u(x) = U(x,s_1)\phi_1 + U(x,s_2)\phi_2$ where $s_1 = -0.5\ell$ and $s_2 = 1.5\ell$. Please find *C* such that $\det[U] = 0$ where $[U]\begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} p \\ q \end{Bmatrix}$ (20%)



Ans:

(1) $[U] = \begin{bmatrix} \frac{\ell}{2} + C & -C \\ C & -\frac{\ell}{2} - C \end{bmatrix}$, $[T] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(2) When *C* is equal to zero, there is no degenerate scale.

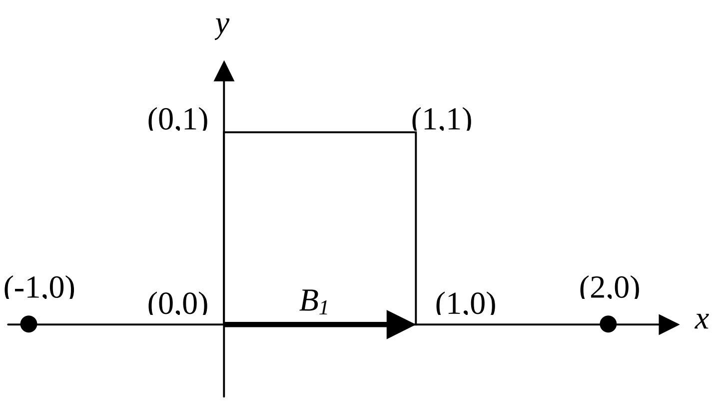
(3) When *C* is equal to $-\frac{1}{4}$, there is degenerate scale where is $\ell = 1$.

(4) $C = -0.5\ell$

4. Determine

(a) $\int_{B_1} U(s,x)dB(s) = ?$ *x*=(-1,0)

(b) $\int_{B_1} T(s,x)dB(s) = ?$ (10%)



Ans: (a) $\int_{B_1} U(s,x)dB(s) = 2\ln 2 - 1$

(b) $\int_{B_1} T(s,x)dB(s) = 0$

5. Please write the five advantages and five disadvantages of BEM.　(10%)

Ans:

Advantages:

(1) 僅須對邊界作離散，而不需對領域作切割，且所需元素較有限元素法少。
(2) 適用於無限域及半無限域問題。
(3) 可只求得有興趣的節點未知量。
(4) 求出結果，不須做微分運算，可得精確與平滑的解。
(5) 適合處理高梯度變化的問題。

Disadvantages:

(1) 退化尺度問題的產生。
(2) 退化邊界問題需特別處理。
(3) 奇異及超奇異積分需特別處理。
(4) 針對內域聲場問題會有假根的產生。
(5) 針對外域聲場問題會有虛擬頻率的產生。

6. What is the rigid body test (constant potential) to determine the diagonal terms of [T] and [M] matrices ? How to apply the technique to degenerate boundary ?　(10%)

Ans:

(1) Rigid body test: 當所考慮的問題的特性矩陣之代數方程需滿足某個特性解時，則可將特性解帶入特性矩陣之代數方程，可求得奇異積分值，不過前提是其他非奇異積分的係數需先求出。

(2) 給予一人工邊界，藉由人工邊界來求得奇異積分的係數。