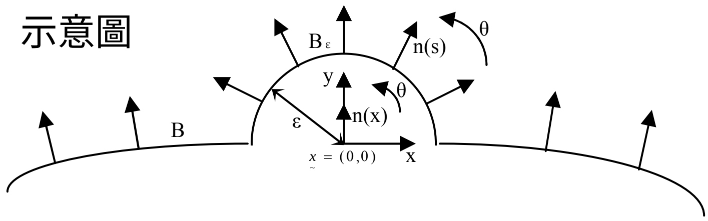
Derivation of dual BIE (including the singularity)

示意圖



Defined：

$$\underset{\sim}{x}=(0,0)\,,\ \underset{\sim}{s}=(\varepsilon\cos\theta,\varepsilon\sin\theta)$$

$$n(x)=\bar{n}=(0,1)\,,\,n(s)=n=(\cos\theta,\sin\theta)$$

$$y_i=x_i-s_i=(-\varepsilon\cos\theta,-\varepsilon\sin\theta)$$

$$u(s)=u(x)+\frac{\partial u(s)}{\partial s_1}\varepsilon\cos\theta+\frac{\partial u(s)}{\partial s_2}\varepsilon\sin\theta$$

$$t(s)=\frac{\partial u(s)}{\partial s_1}\cos\theta+\frac{\partial u(s)}{\partial s_2}\sin\theta$$

$$U(s,x)=\ln r=\ln\varepsilon$$

$$\int_B U(s,x)t(s)dB(s)=\varepsilon\ln\varepsilon\int_0^{\pi}\left(\frac{\partial u(s)}{\partial s_1}\cos\theta+\frac{\partial u(s)}{\partial s_2}\sin\theta\right)\Big|_{s=x}d\theta=0\quad(\varepsilon\to 0)$$

$$T(s,x)=-\frac{y_in_i}{r^2}=\frac{1}{\varepsilon}$$

$$\int_B T(s,x)u(s)dB(s)=\int_0^{\pi}\frac{1}{\varepsilon}\left(u(x)+\frac{\partial u(s)}{\partial s_1}\varepsilon\cos\theta+\frac{\partial u(s)}{\partial s_2}\varepsilon\sin\theta\right)\Big|_{s=x}\varepsilon d\theta=\pi u(x)\quad(\varepsilon\to 0)$$

域內點邊界積分方程式

$$2\pi u(x)=\int_B T(s,x)u(s)dB(s)-\int_B U(s,x)t(s)dB(s)$$

$$2\pi u(x)=C.P.V.\int_B T(s,x)u(s)dB(s)+\pi u(x)-R.P.V.\int_B U(s,x)t(s)dB(s)$$

$$\Rightarrow\pi u(x)=C.P.V.\int_B T(s,x)u(s)dB(s)-C.P.V.\int_B U(s,x)t(s)dB(s)$$

$$L(s,x)=\frac{y_i\bar{n}_i}{r^2}=-\frac{\sin\theta}{\varepsilon}$$

$$\int_B L(s,x)t(s)dB(s)=\int_0^{\pi}\left(-\frac{\sin\theta}{\varepsilon}\right)\left(\frac{\partial u(s)}{\partial s_1}\cos\theta+\frac{\partial u(s)}{\partial s_2}\sin\theta\right)\Big|_{s=x}\varepsilon d\theta=-\frac{\pi}{2}t(s)$$

$$M(s,x)=\frac{2y_iy_jn_i\bar{n}_j}{r^4}-\frac{n_i\bar{n}_i}{r^2}=\frac{\sin\theta}{\varepsilon^2}$$

$$\int_B M(s,x)u(s)dB(s)=\int_0^{\pi}\frac{\sin\theta}{\varepsilon^2}\left(u(x)+\frac{\partial u(s)}{\partial s_1}\varepsilon\cos\theta+\frac{\partial u(s)}{\partial s_2}\varepsilon\sin\theta\right)\Big|_{s=x}\varepsilon d\theta=\frac{2}{\varepsilon}u(x)+\frac{\pi}{2}t(s)$$

域內點邊界積分方程式

$$2\pi t(x)=\int_B M(s,x)u(s)dB(s)-\int_B L(s,x)t(s)dB(s)$$

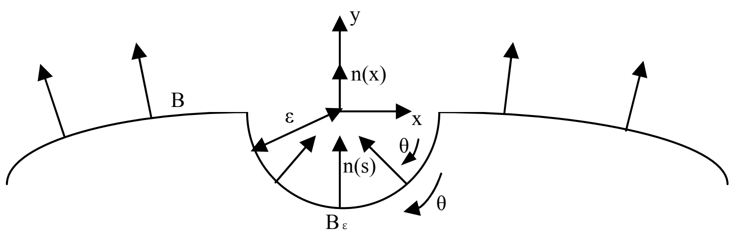
$$2\pi t(x)=H.P.V.\int_B M(s,x)u(s)dB(s)+\frac{\pi}{2}t(s)-C.P.V.\int_B L(s,x)t(s)dB(s)+\frac{\pi}{2}t(s)$$

$$\Rightarrow\pi t(x)=H.P.V.\int_B M(s,x)u(s)dB(s)-C.P.V.\int_B L(s,x)t(s)dB(s)$$

$$=C.P.V.\int_B M(s,x)u(s)dB(s)+\frac{2}{\varepsilon}u(x)-C.P.V.\int_B L(s,x)t(s)dB(s)$$

Derivation of dual BIE (excluding the singularity)

示意圖



Defined：

$$\underset{\sim}{x}=(0,0)\,,\,\underset{\sim}{s}=(\varepsilon\cos\theta,-\varepsilon\sin\theta)\,,0<\theta<\pi$$

$$n(x)=\bar{n}=(0,1)\,,n(s)=n=(-\cos\theta,\sin\theta)$$

$$y_i=x_i-s_i=(-\varepsilon\cos\theta,\varepsilon\sin\theta)$$

$$u(s)=u(x)+\frac{\partial u(s)}{\partial s_1}\varepsilon\cos\theta-\frac{\partial u(s)}{\partial s_2}\varepsilon\sin\theta$$

$$t(s)=-\frac{\partial u(s)}{\partial s_1}\cos\theta+\frac{\partial u(s)}{\partial s_2}\sin\theta$$

$$U(s,x)=\ln r=\ln\varepsilon$$

$$\int_B U(s,x)t(s)dB(s)=\varepsilon\ln\varepsilon\int_0^\pi(-\frac{\partial u(s)}{\partial s_1}\cos\theta+\frac{\partial u(s)}{\partial s_2}\sin\theta)\Big|_{s=x}d\theta=0\quad(\varepsilon\to0)$$

$$T(s,x)=-\frac{y_in_i}{r^2}=-\frac{1}{\varepsilon}$$

$$\int_B T(s,x)u(s)dB(s)=\int_0^\pi-\frac{1}{\varepsilon}(u(x)+\frac{\partial u(s)}{\partial s_1}\varepsilon\cos\theta-\frac{\partial u(s)}{\partial s_2}\varepsilon\sin\theta)\Big|_{s=x}\varepsilon d\theta=-\pi u(x)\quad(\varepsilon\to0)$$

域外點邊界積分方程式

$$0=\int_B T(s,x)u(s)dB(s)-\int_B U(s,x)t(s)dB(s)$$

$$0=C.P.V.\int_B T(s,x)u(s)dB(s)-\pi u(x)-R.P.V.\int_B U(s,x)t(s)dB(s)$$

$$\Rightarrow\pi u(x)=C.P.V.\int_B T(s,x)u(s)dB(s)-C.P.V.\int_B U(s,x)t(s)dB(s)$$

$$L(s,x)=\frac{y_i\bar{n}_i}{r^2}=\frac{\sin\theta}{\varepsilon}$$

$$\int_B L(s,x)t(s)dB(s)=\int_0^\pi\frac{\sin\theta}{\varepsilon}(-\frac{\partial u(s)}{\partial s_1}\cos\theta+\frac{\partial u(s)}{\partial s_2}\sin\theta)\Big|_{s=x}\varepsilon d\theta=\frac{\pi}{2}t(s)$$

$$M(s,x)=\frac{2y_iy_jn_i\bar{n}_j}{r^4}-\frac{n_i\bar{n}_i}{r^2}=\frac{\sin\theta}{\varepsilon^2}$$

$$\int_B M(s,x)u(s)dB(s)=\int_0^\pi\frac{\sin\theta}{\varepsilon^2}(u(x)+\frac{\partial u(s)}{\partial s_1}\varepsilon\cos\theta-\frac{\partial u(s)}{\partial s_2}\varepsilon\sin\theta)\Big|_{s=x}\varepsilon d\theta=\frac{2}{\varepsilon}u(x)-\frac{\pi}{2}t(s)$$

域外點邊界積分方程式

$$0=\int_B M(s,x)u(s)dB(s)-\int_B L(s,x)t(s)dB(s)$$

$$0=H.P.V.\int_B M(s,x)u(s)dB(s)-\frac{\pi}{2}t(s)-C.P.V.\int_B L(s,x)t(s)dB(s)-\frac{\pi}{2}t(s)$$

$$\Rightarrow\pi t(x)=H.P.V.\int_B M(s,x)u(s)dB(s)-C.P.V.\int_B L(s,x)t(s)dB(s)$$

$$=C.P.V.\int_B M(s,x)u(s)dB(s)+\frac{2}{\varepsilon}u(x)-C.P.V.\int_B L(s,x)t(s)dB(s)$$

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