

Double transformation

1. Real operation:

$$R^4 c = c$$

$$R = 1, -1, i, -i$$

2. Matrix operation:

$$A^4 \mathbf{x} = \mathbf{x}$$

3. Householder matrix:

$$H^2 \mathbf{x} = \mathbf{x}$$

4. Fourier transform:

$$\mathcal{F}(\mathcal{F}(f(t))) = \mathcal{F}^2\{f(t)\} = f(-t)$$

$$\mathcal{F}^4\{f(t)\} = f(t)$$

5. Laplace transform of Euler-Cauchy equation:

$$t^2 \ddot{y}(t) - 4t \dot{y}(t) + 6y(t) = 0 \rightarrow s^2 \ddot{Y}(s) - 4s \dot{Y}(s) + 6Y(s) = 0$$

$$s^2 \ddot{Y}(s) + 8s \dot{Y}(s) + 12Y(s) = 0 \rightarrow t^2 \ddot{y}(t) - 4t \dot{y}(t) + 6y(t) = 0$$

6. Hilbert transform:

$$\mathcal{H}(\mathcal{H}(y(t))) = -y(t)$$

7. Hypersingular operator: (Laplace)

$$\int_B M(\theta, \phi) u(\theta) dB(\theta) = \mathcal{M}(u(\theta))$$

$$\{\mathcal{M}\{\mathcal{M}(u(\theta))\}\} = -u''(\theta)$$

Pseudo-differential operator 1.

8. Hypersingular operator: (Helmholtz)

$$\int_B M(\theta, \phi) u(\theta) dB(\theta) = \mathcal{M}(u(\theta))$$

$$\{\mathcal{M}\{\mathcal{M}(u(\theta))\}\} = ?$$

9. Weakly singular operator: (Laplace)

$$\int_B U(\theta, \phi) u(\theta) dB(\theta) = \mathcal{U}(u(\theta))$$

$$\{\mathcal{U}\{\mathcal{U}(u(\theta))\}\} = ?$$

Pseudo-differential operator -1.

10. Weakly singular operator: (Helmholtz)

$$\int_B U(\theta, \phi) u(\theta) dB(\theta) = \mathcal{U}(u(\theta))$$

$$\{\mathcal{U}\{\mathcal{U}(u(\theta))\}\} = ?$$