Part III

Plate vibration

Outlines

Outlines

1. Introduction
2. Plate vibration
3. Derivation by Circulants
4. SVD updating terms
5. Conclusions

Governing Equation:

$$\nabla^4 u(x) = \lambda^4 u(x),\ x \in \Omega$$

$$\lambda^4 = \frac{\omega^2 \rho h}{D}$$

$$D = \frac{E h^3}{12(1-\nu^2)}$$

ω is the angle frequency

harmonic operator

ρ is the surface density

eral displacement

D is the flexural rigidity

equency parameter

h is the plates thickness

E is the Young's modulus

main of the thin plates

ν is the Poisson ration

Field representation using RBF

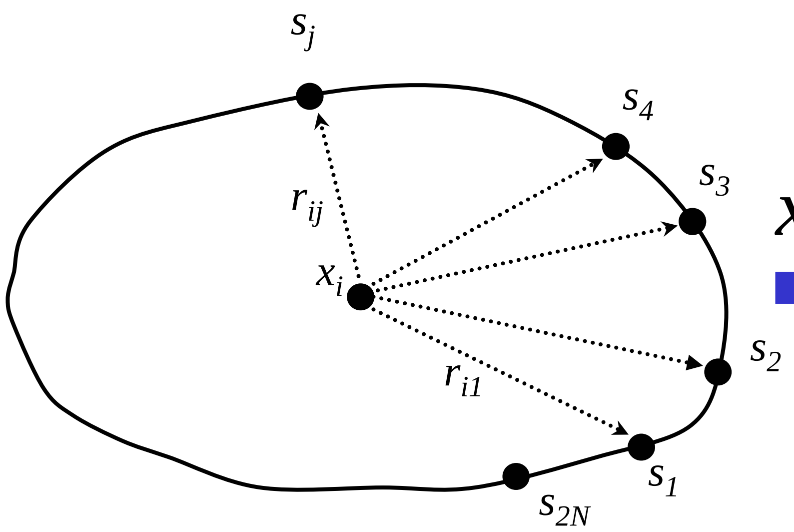
$$u(x) = \sum c_j \psi(x_i, s_j)$$

$$\psi(x, s) = \psi(r)$$

$$\{u\} = \begin{bmatrix} \ddots & * & * & * & * \\ * & \ddots & * & * & * \\ * & * & \ddots & * & * \\ * & * & * & \ddots & * \\ * & * & * & * & \ddots \end{bmatrix} \{c\}$$

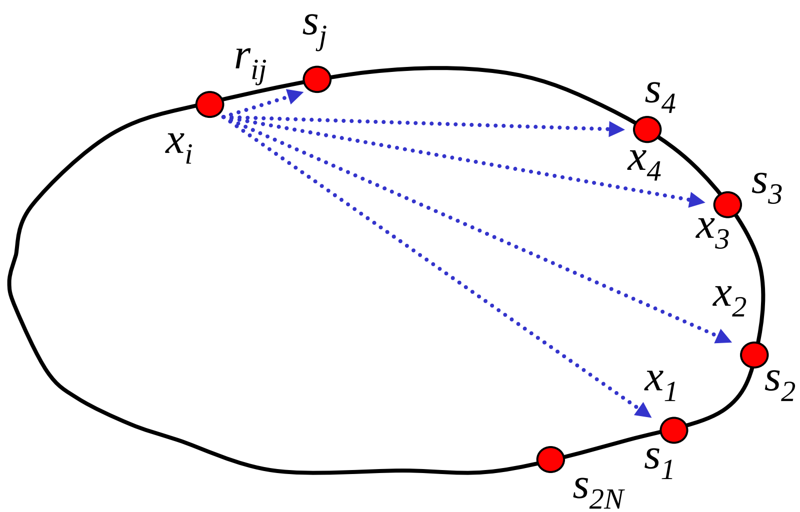
$r \neq 0$

$r = 0$



Field representation

$$x \rightarrow B$$

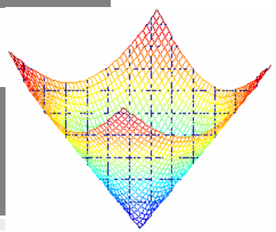


To match B.C.

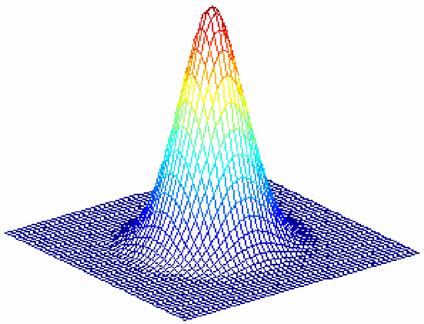
Data bank of RBF

Radial basis function (RBFs)

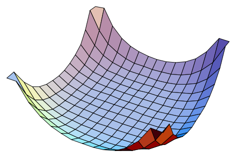
- **Mesh method**
  - **Globally-supported RBFs**
    - **DRBEM**
      Nardini
      Brebbia
      1982
      $\psi(r) = 1 + r$
    - **Method of particular integral**
      Ahmad & Banerjee
      1986
      $\psi(r) = C - r$
    - → **Equivalence**
      Polyzos & Beskos
      1994



- **Meshless method**
  - **Compactly-supported RBFs**
    C.S. Chen
    $\psi(r) = (1-r)_+^4 (4r+1)$



  - **Globally-supported RBFs**
    - **Volume potential**
      → **Method of fundamental solution**
      $\psi(r) = U_c(s,x) = U_c(|x-s|) = U_c(r)$
      (Potential theory)
    - **The present method**
      → **Imaginary-part fundamental solution**
      $\psi(r) = J_0(\lambda r) + I_0(\lambda r)$



Outlines

1. Introduction
2. Plate vibration
3. Derivation by Circulants
4. SVD updating terms
5. Conclusions

Displacement field of plate vibration

$$u(s,x)=\sum_{j=1}^{2N}P(s_j,x)\phi_j+\sum_{j=1}^{2N}Q(s_j,x)\psi_j$$

2*N* is the number of boundary nodes

***s* is the source point**

***x* is the collocation point**

ϕ_j and ψ_j are the unknown densities

P* and *Q* can be obtained from either two combinations of *U*, θ, *M* and *V

Four kernels

Imaginary-part fundamental solution

$$U(s,x) = \mathrm{Im}\{\frac{i}{8\lambda^2}(H_0^{(2)}(\lambda r) + H_0^{(1)}(i\lambda r))\}$$

$$U(s,x) = \frac{1}{8\lambda^2}(J_0(\lambda r) + I_0(\lambda r))$$

Four kernels

$$\Theta(s,x) = K_\theta(U(s,x))$$

$$M(s,x) = K_m(U(s,x))$$

$$V(s,x) = K_v(U(s,x))$$

Operators

$$K_\theta(\cdot) = \frac{\partial(\cdot)}{\partial n}$$

$$K_m(\cdot) = \nu \nabla^2(\cdot) + (1-\nu)\frac{\partial^2(\cdot)}{\partial n^2}$$

$$K_v(\cdot) = \frac{\partial \nabla^2(\cdot)}{\partial n} + (1-\nu)\frac{\partial}{\partial t}\left(\frac{\partial^2(\cdot)}{\partial n \partial t}\right)$$

Slope, Moment and Shear

Slope $$\theta(x) = K_{\theta}(u(x))$$

Moment $$m(x) = K_{m}(u(x))$$

Shear $$v(x) = K_{v}(u(x))$$

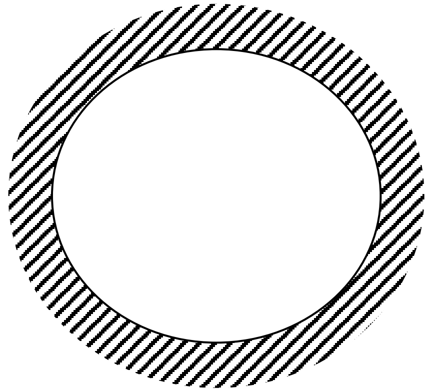
True eigenequation of three cases

	B.C.	True eigenequation
Clamped plate	$u(x)=0$ $\square(x)=0$	$J'_\ell(\lambda\rho)I_\ell(\lambda\rho)-I'_\ell(\lambda\rho)J_\ell(\lambda\rho)=0$
Simply-supported plate	$u(x)=0$ $m(x)=0$	$\frac{I_{\ell+1}(\lambda\rho)}{I_\ell(\lambda\rho)}+\frac{J_{\ell+1}(\lambda\rho)}{J_\ell(\lambda\rho)}=\frac{2\lambda\rho}{(1-\nu)}$
Free plate	$m(x)=0$ $v(x)=0$	$(\ell^2(\ell^2-1)(-1+\nu)^2+\lambda^4a^4)(J_{\ell+1}(\lambda a)I_\ell(\lambda a)+J_\ell(\lambda a)I_{\ell+1}(\lambda a))$ $+2\ell\lambda^2a^2(1-\ell)(-1+\nu)(J_{\ell+1}(\lambda a)I_\ell(\lambda a)-J_\ell(\lambda a)I_{\ell+1}(\lambda a))$ $+\lambda a(-1+\nu)(2\lambda^2a^2J_{\ell+1}(\lambda a)I_{\ell+1}(\lambda a)+4\ell^2(-1+\ell)J_\ell(\lambda a)I_\ell(\lambda a)=0$

Clamped plate

$$u(x)=0 \Rightarrow [U]\{\phi\}+[\Theta]\{\psi\}=\{0\}$$

$$\theta(x)=0 \Rightarrow [U_\theta]\{\phi\}+[\Theta_\theta]\{\psi\}=\{0\}$$



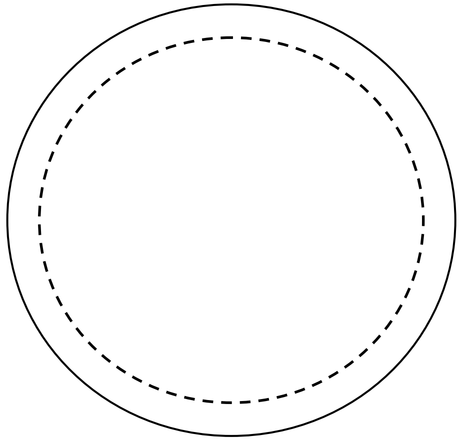
Nontrivial

$$0=\det\left[SM^{C}\right] \Leftarrow \begin{bmatrix} U & \Theta \\ U_\theta & \Theta_\theta \end{bmatrix}\begin{Bmatrix}\phi \\ \psi\end{Bmatrix}=\begin{Bmatrix}0 \\ 0\end{Bmatrix}$$

Simply-supported plate

$$u(x)=0 \Rightarrow [U]\{\phi\}+[\Theta]\{\psi\}=\{0\}$$

$$m(x)=0 \Rightarrow [U_m]\{\phi\}+[\Theta_m]\{\psi\}=\{0\}$$



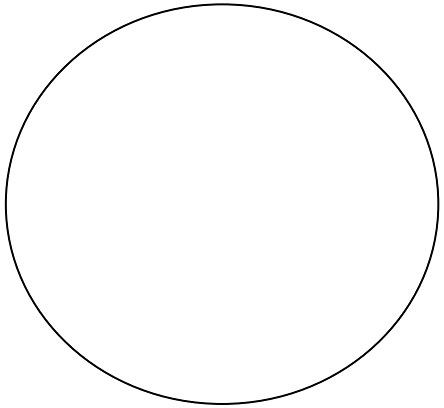
Nontrivial

$$0=\left[SM^{S}\right] \Leftarrow \begin{bmatrix} U & \Theta \\ U_m & \Theta_m \end{bmatrix}\begin{Bmatrix}\phi \\ \psi\end{Bmatrix}=\begin{Bmatrix}0 \\ 0\end{Bmatrix}$$

Free plate

$$m(x) = 0 \Rightarrow [U_m]\{\phi\} + [\Theta_m]\{\psi\} = \{0\}$$

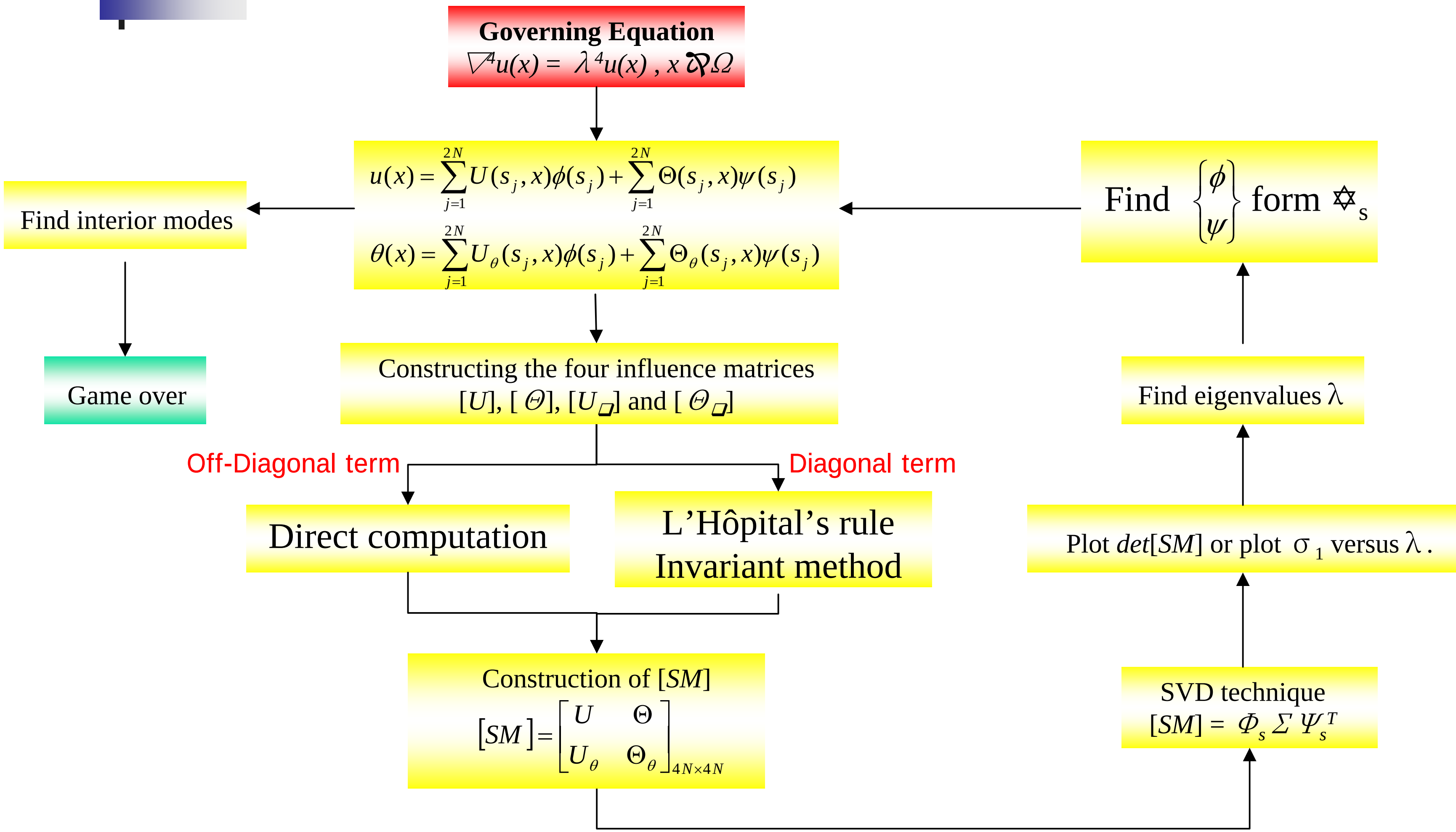
$$v(x) = 0 \Rightarrow [U_v]\{\phi\} + [\Theta_v]\{\psi\} = \{0\}$$



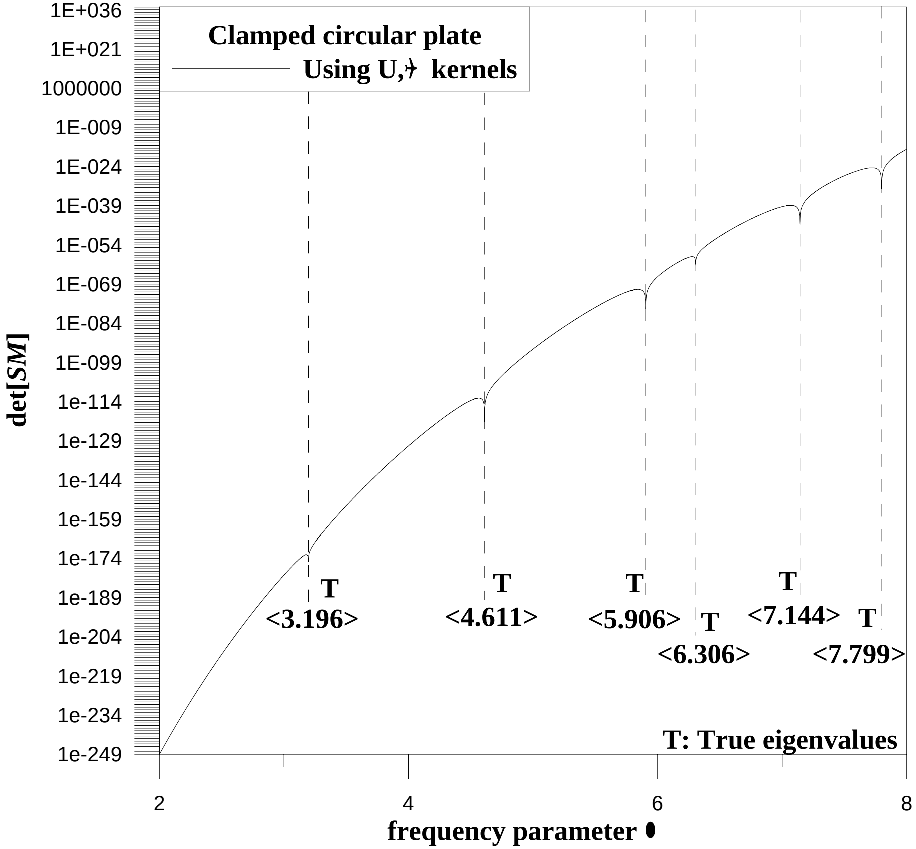
Nontrivial

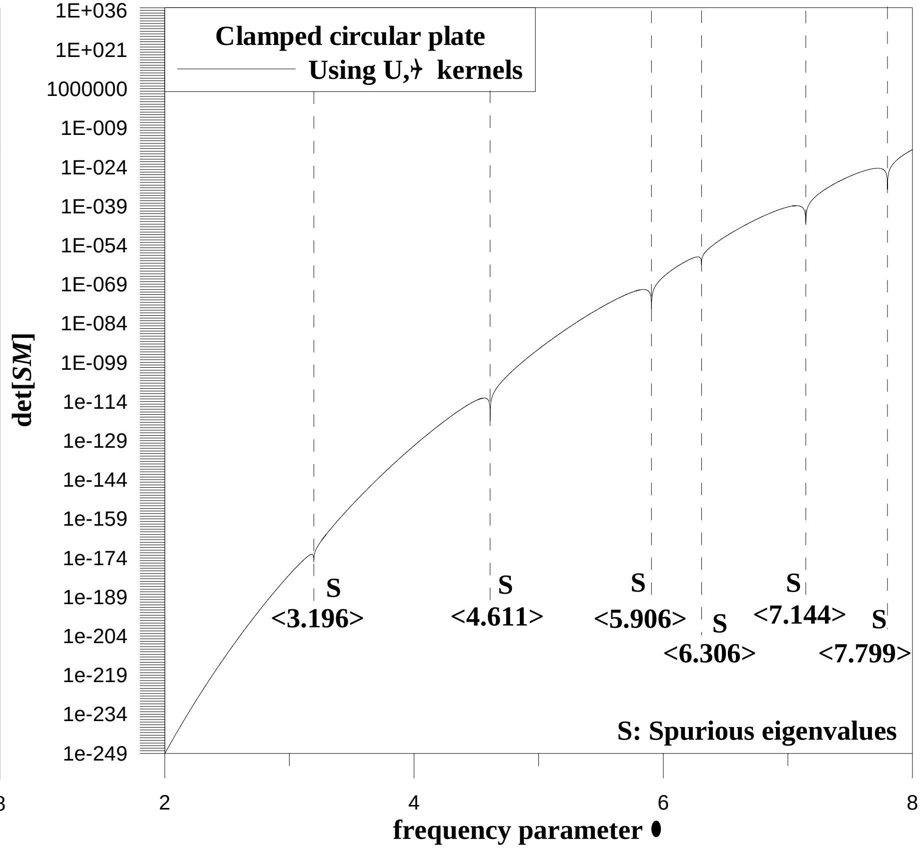
$$0 = \left[SM^F\right] \Leftarrow \begin{bmatrix} U_m & \Theta_m \\ U_v & \Theta_v \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Flow chart of the present method for clamped case by using U and ✈ kernels

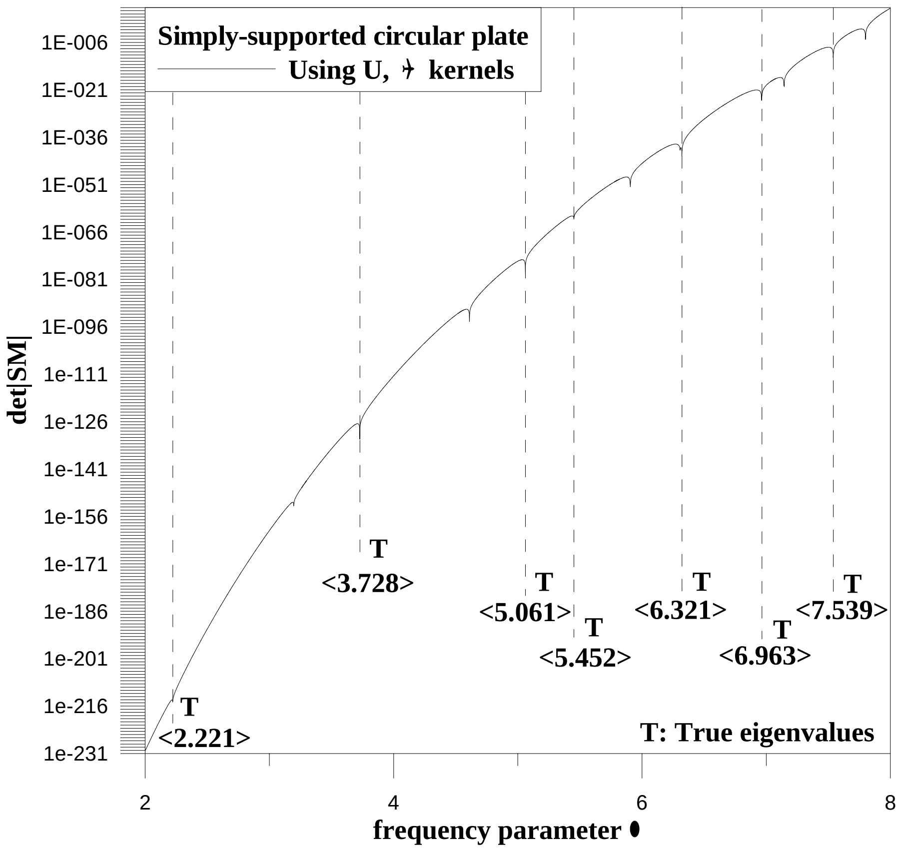


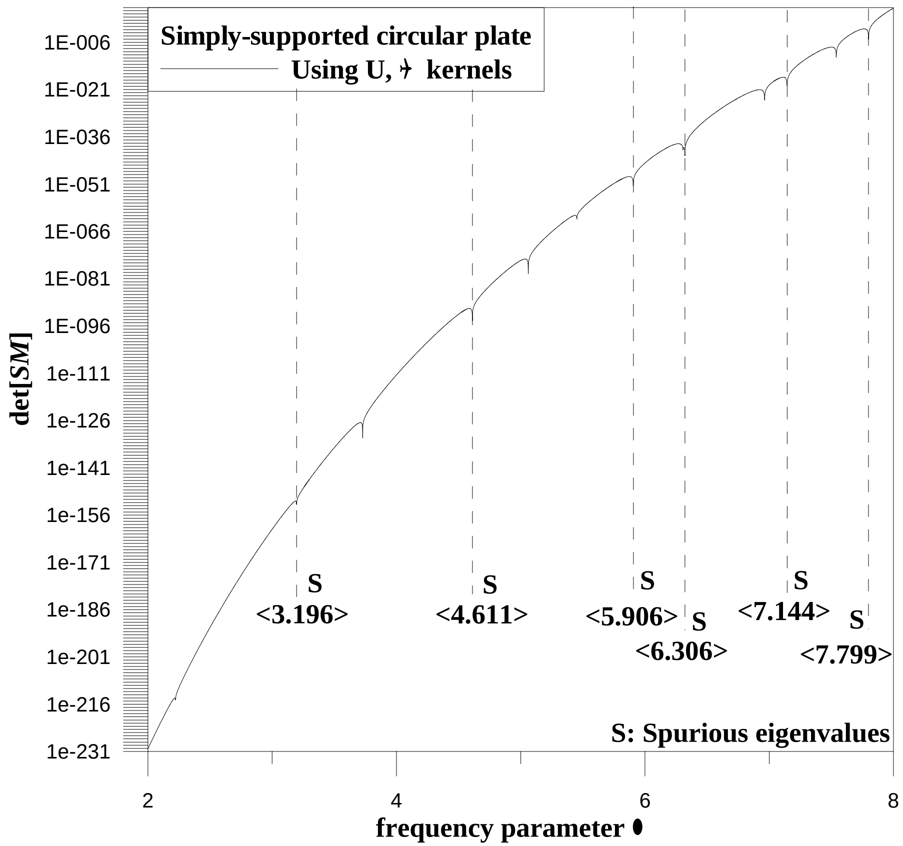
Case 1: Circular clamped plate using the present method



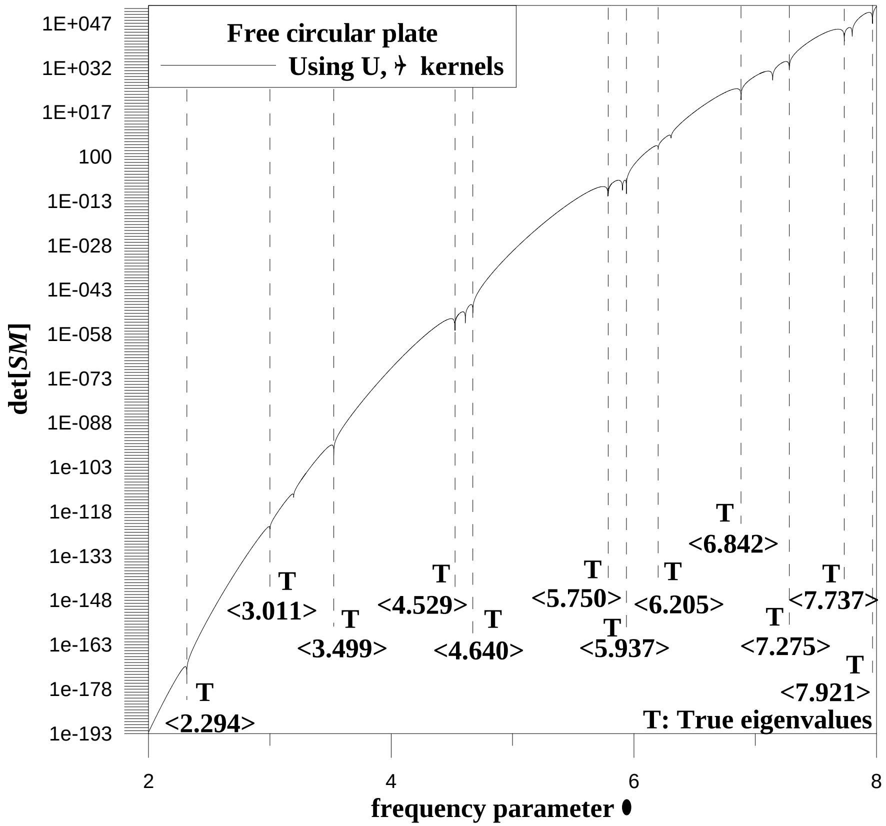


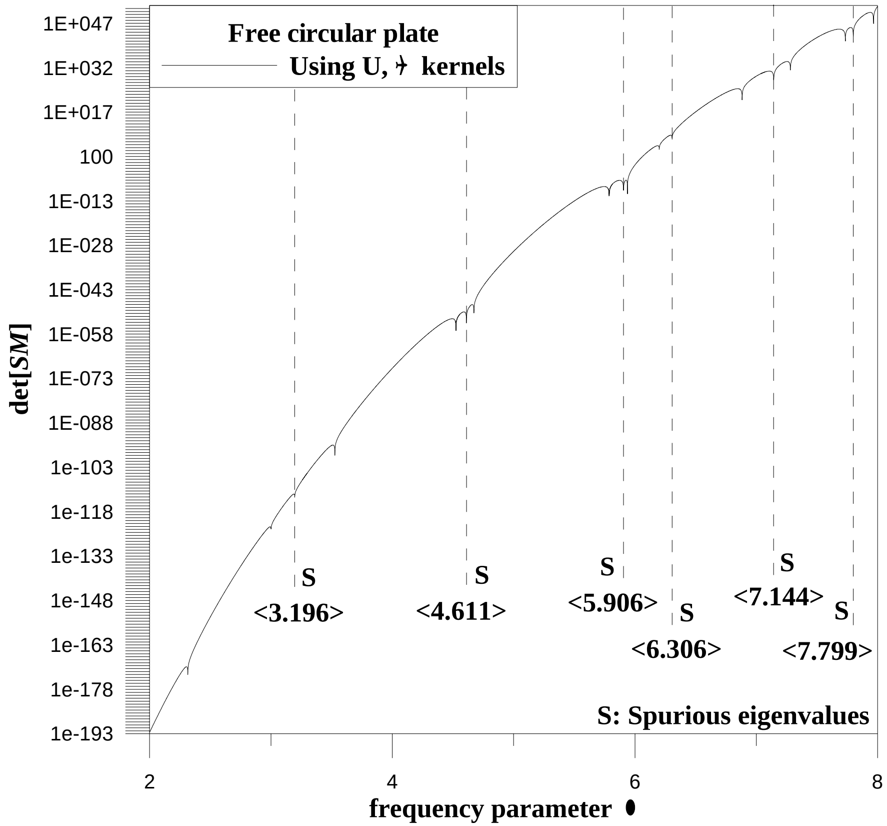
Case 2: Circular simply-supported plate using the present method



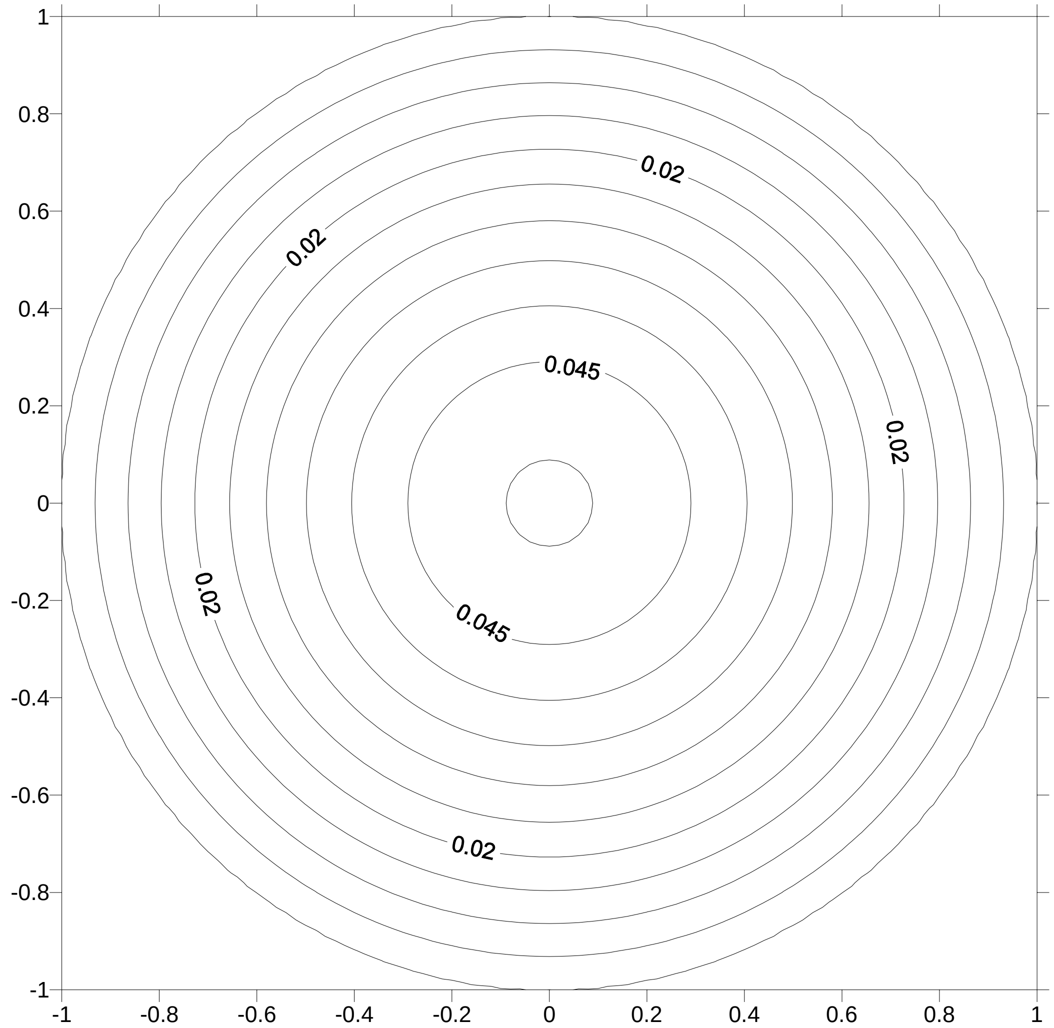


Case 3: Circular free plate using the present method



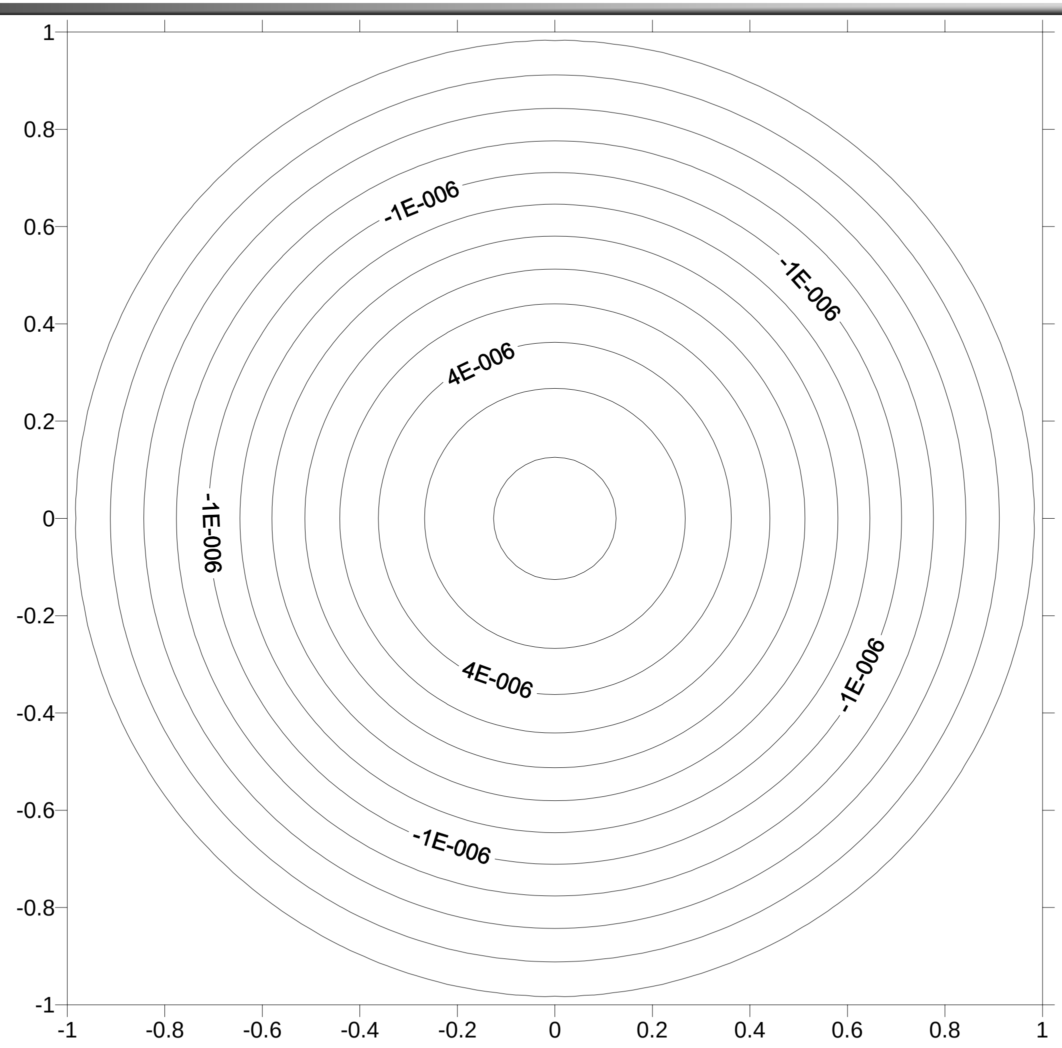


Mode 1 (λ =2.221)



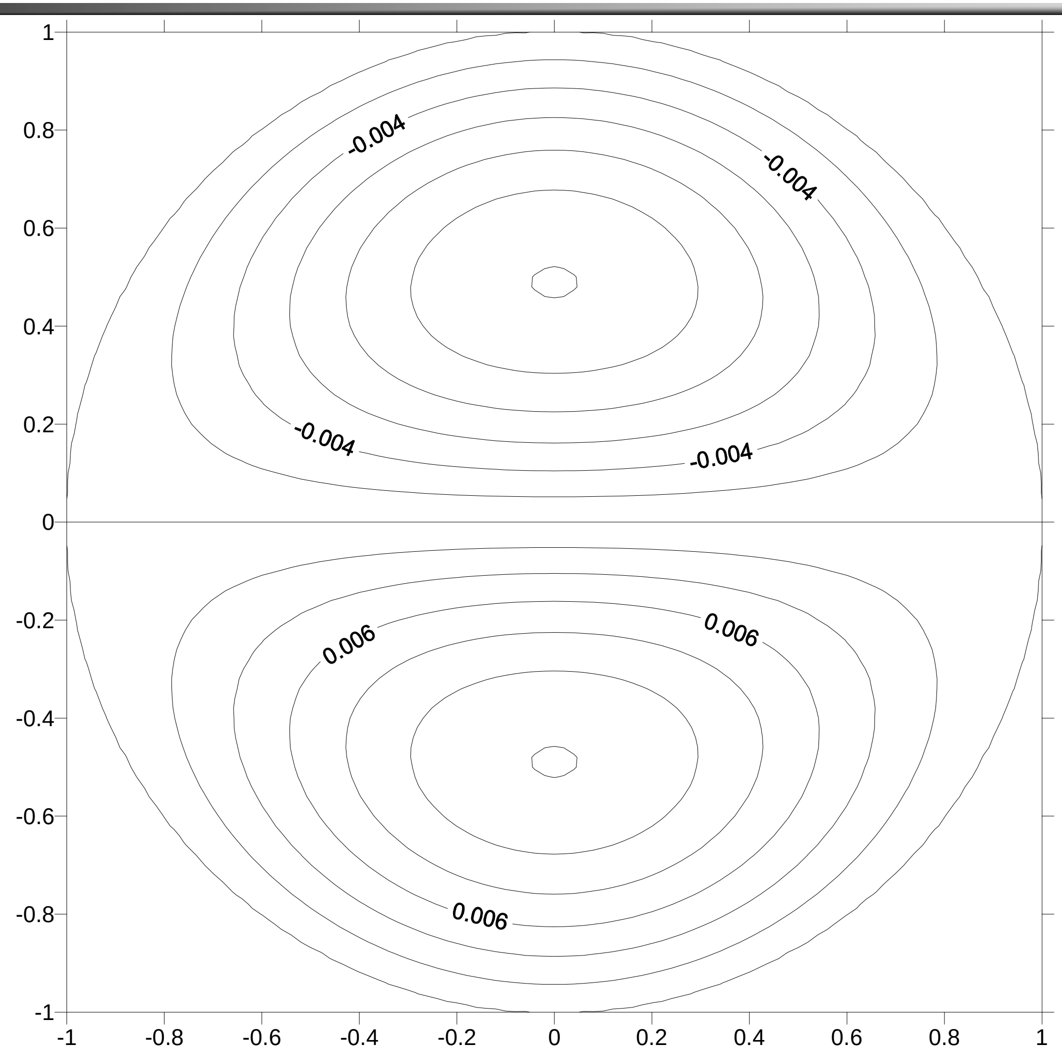
0.02
0.02
0.045
0.02
0.02
0.045
0.02
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Mode 2 (λ =3.196)



-1E-006
-1E-006
4E-006
-1E-006
4E-006
-1E-006
-1E-006
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Mode 3 (λ =3.728)



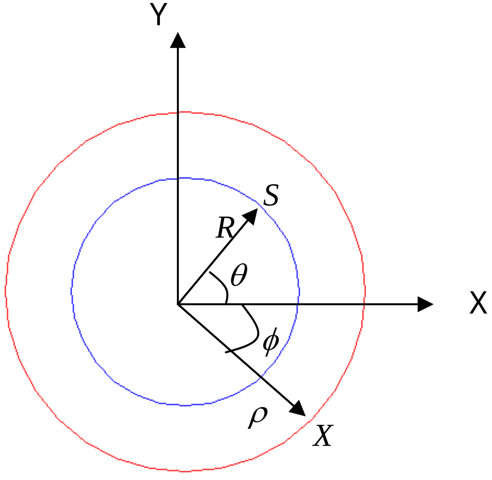
-0.004
-0.004
-0.004
-0.004
0.006
0.006
0.006
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Outlines

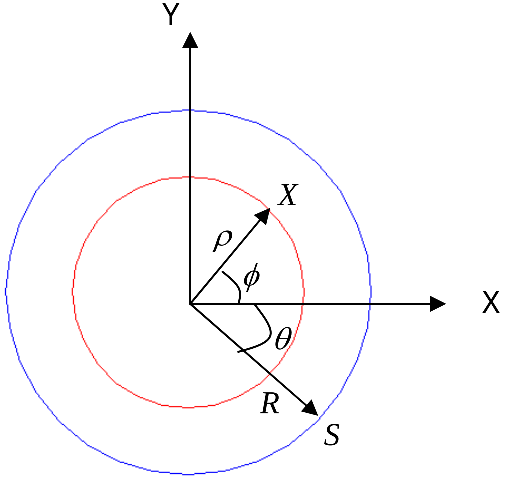
1. Introduction
2. Plate vibration
3. Derivation by Circulants
4. SVD updating terms
5. Conclusions

Degenerate kernels for circular case

The degenerate kernels for interior and exterior problems:



Exterior problem



Interior problem

Blue: field points
Red: source points

$$U(s,x)=\begin{cases} U^{I}(R,\theta;\ \rho,\phi)=\dfrac{1}{8\lambda^{2}}\displaystyle\sum_{m=-\infty}^{\infty}[J_{m}(\lambda R)J_{m}(\lambda\rho)+(-1)^{m}I_{m}(\lambda R)I_{m}(\lambda\rho)]\big(\cos(m(\theta-\phi))\big), & R>\rho \\ U^{E}(R,\theta;\ \rho,\phi)=\dfrac{1}{8\lambda^{2}}\displaystyle\sum_{m=-\infty}^{\infty}[J_{m}(\lambda\rho)J_{m}(\lambda R)+(-1)^{m}I_{m}(\lambda\rho)I_{m}(\lambda R)]\big(\cos(m(\theta-\phi))\big), & R<\rho \end{cases}$$

Circulants

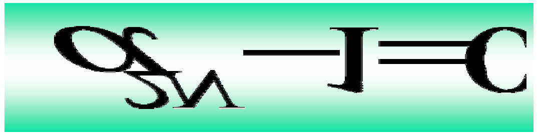
$$[K]=\begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-=3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_0 & a_1 \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix}$$

$$a_{j-i}=K(s_j,x_i)$$

$$[K]=a_0 I+a_1(C_{2N})^1+a_2(C_{2N})^2+\cdots+a_{2N-1}(C_{2N})^{2N-1}$$

Circulants

$$C_{2N} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{2N \times 2N}$$



$$\alpha_\ell = e^{i\frac{2\pi\ell}{2N}} = \cos\left(\frac{2\pi\ell}{2N}\right) + i\sin\left(\frac{2\pi\ell}{2N}\right)$$

Circulants

$$\lambda_\ell^{[U]} = a_0 + a_1\alpha_\ell + a_2\alpha_\ell^2 + \cdots + a_{2N-1}\alpha_\ell^{2N-1}$$

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

$$\lambda_\ell^{[U]} = \frac{N}{4\lambda^2}[J_\ell(\lambda\rho)J_\ell(\lambda\rho) + (-1)^\ell I_\ell(\lambda\rho)I_\ell(\lambda\rho)]$$

The eigenvalues of matrices

$$[U] = \Phi \Sigma_U \Phi^T$$

$$= \Phi \begin{bmatrix} \lambda_0^{[U]} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \lambda_1^{[U]} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \lambda_{-1}^{[U]} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{(N-1)}^{[U]} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{-(N-1)}^{[U]} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \lambda_N^{[U]} \end{bmatrix}_{2N \times 2N} \Phi^T$$

Relationships

$$[U] \xrightarrow{SVD} \Sigma_U \rightarrow \lambda_\ell^{[U]}$$

$$[\Theta] \xrightarrow{SVD} \Sigma_\Theta \rightarrow \lambda_\ell^{[\Theta]}$$

$$[U_\theta] \xrightarrow{SVD} \Sigma_{U_\theta} \rightarrow \mu_\ell^{[U]}$$

$$[\Theta_\theta] \xrightarrow{SVD} \Sigma_{\Theta_\theta} \rightarrow \mu_\ell^{[\Theta]}$$

Diagonal matrix

Diagonal element

Determinant (for clamped)

$$\left[SM^{C}\right]=\begin{bmatrix}\Phi\Sigma_{U}\Phi^{T} & \Phi\Sigma_{\Theta}\Phi^{T}\\ \Phi\Sigma_{U_\theta}\Phi^{T} & \Phi\Sigma_{\Theta_\theta}\Phi^{T}\end{bmatrix}_{2N\times 2N}$$

$$=\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_{U} & \Sigma_{\Theta}\\ \Sigma_{U_\theta} & \Sigma_{\Theta_\theta}\end{bmatrix}\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}^{T}$$

Eigenequation for clamped boundary

$$\det[SM^C]$$

$$= \prod_{\ell=-(N+1)}^{N} (\lambda_\ell^{[U]} \mu_\ell^{[\Theta]} - \lambda_\ell^{[\Theta]} \mu_\ell^{[U]})$$

$$= \prod_{\ell=-(N+1)}^{N} \frac{(-1)^\ell N^2}{16\lambda^2} [J_{\ell+1}(\lambda\rho) I_\ell(\lambda\rho) + I_{\ell+1}(\lambda\rho) J_\ell(\lambda\rho)]$$

$$\{J_{\ell+1}(\lambda\rho) I_\ell(\lambda\rho) + I_{\ell+1}(\lambda\rho) J_\ell(\lambda\rho)\} = 0,$$

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

Determinant (for simply-supported)

$$\left[SM^{S}\right]=\begin{bmatrix}\Phi\Sigma_{U}\Phi^{T} & \Phi\Sigma_{\Theta}\Phi^{T}\\ \Phi\Sigma_{U_m}\Phi^{T} & \Phi\Sigma_{\Theta_m}\Phi^{T}\end{bmatrix}$$

$$=\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_{U} & \Sigma_{\Theta}\\ \Sigma_{U_m} & \Sigma_{\Theta_m}\end{bmatrix}\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}^{T}$$

Relationships

$$[U] \xrightarrow{SVD} \Sigma_U \rightarrow \lambda_\ell^{[U]}$$

$$[\Theta] \xrightarrow{SVD} \Sigma_\Theta \rightarrow \lambda_\ell^{[\Theta]}$$

$$[U_m] \xrightarrow{SVD} \Sigma_{U_m} \rightarrow \nu_\ell^{[U]}$$

$$[\Theta_m] \xrightarrow{SVD} \Sigma_{\Theta_m} \rightarrow \nu_\ell^{[\Theta]}$$

Eigenequation for simply-supported boundary

$$\det[SM^S]$$

$$= \prod_{\ell=-(N+1)}^{N} (\lambda_\ell^{[U]} \nu_\ell^{[\Theta]} - \lambda_\ell^{[\Theta]} \nu_\ell^{[U]})$$

$$= \prod_{\ell=-(N+1)}^{N} \frac{(-1)^\ell N^2}{16\lambda^2 \rho} [J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho) J_{\ell+1}(\lambda\rho)]$$

$$\{(-1+\nu)(J_{\ell+1}(\lambda\rho) I_\ell(\lambda\rho) + J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho))$$

$$+ 2\lambda\rho J_\ell(\lambda\rho) I_\ell(\lambda\rho)\} = 0$$

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

Determinant (for simply-supported)

$$\left[SM^F\right]=\begin{bmatrix}\Phi\Sigma_{U_m}\Phi^T & \Phi\Sigma_{\Theta_m}\Phi^T \\ \Phi\Sigma_{U_v}\Phi^T & \Phi\Sigma_{\Theta_v}\Phi^T\end{bmatrix}$$

$$=\begin{bmatrix}\Phi & 0 \\ 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_{U_m} & \Sigma_{U_m} \\ \Sigma_{U_v} & \Sigma_{\Theta_v}\end{bmatrix}\begin{bmatrix}\Phi & 0 \\ 0 & \Phi\end{bmatrix}^T$$

Relationships

$$[U_m] \xrightarrow{SVD} \Sigma_{U_m} \to \nu_\ell^{[U]}$$

$$[\Theta_m] \xrightarrow{SVD} \Sigma_{\Theta_m} \to \nu_\ell^{[\Theta]}$$

$$[U_v] \xrightarrow{SVD} \Sigma_{U_v} \to \delta_\ell^{[U]}$$

$$[\Theta_v] \xrightarrow{SVD} \Sigma_{\Theta_v} \to \delta_\ell^{[\Theta]}$$

Eigenequation for free boundary

$$\det[SM^F]$$

$$= \prod_{\ell=-(N+1)}^{N} (v_\ell^{[U]}\delta_\ell^{[\Theta]} - v_\ell^{[\Theta]}\delta_\ell^{[U]})$$

$$= \prod_{\ell=-(N+1)}^{N} \frac{(-1)^\ell N^2}{16\lambda^2\rho^4}[J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)+J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho)]$$

$$\{\ell^2(\ell^2-1)(-1+v)^2+\lambda^4\rho^4)(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)+J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))$$

$$+2\ell\lambda^2\rho^2(1-\ell)(-1+v)(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)-J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))$$

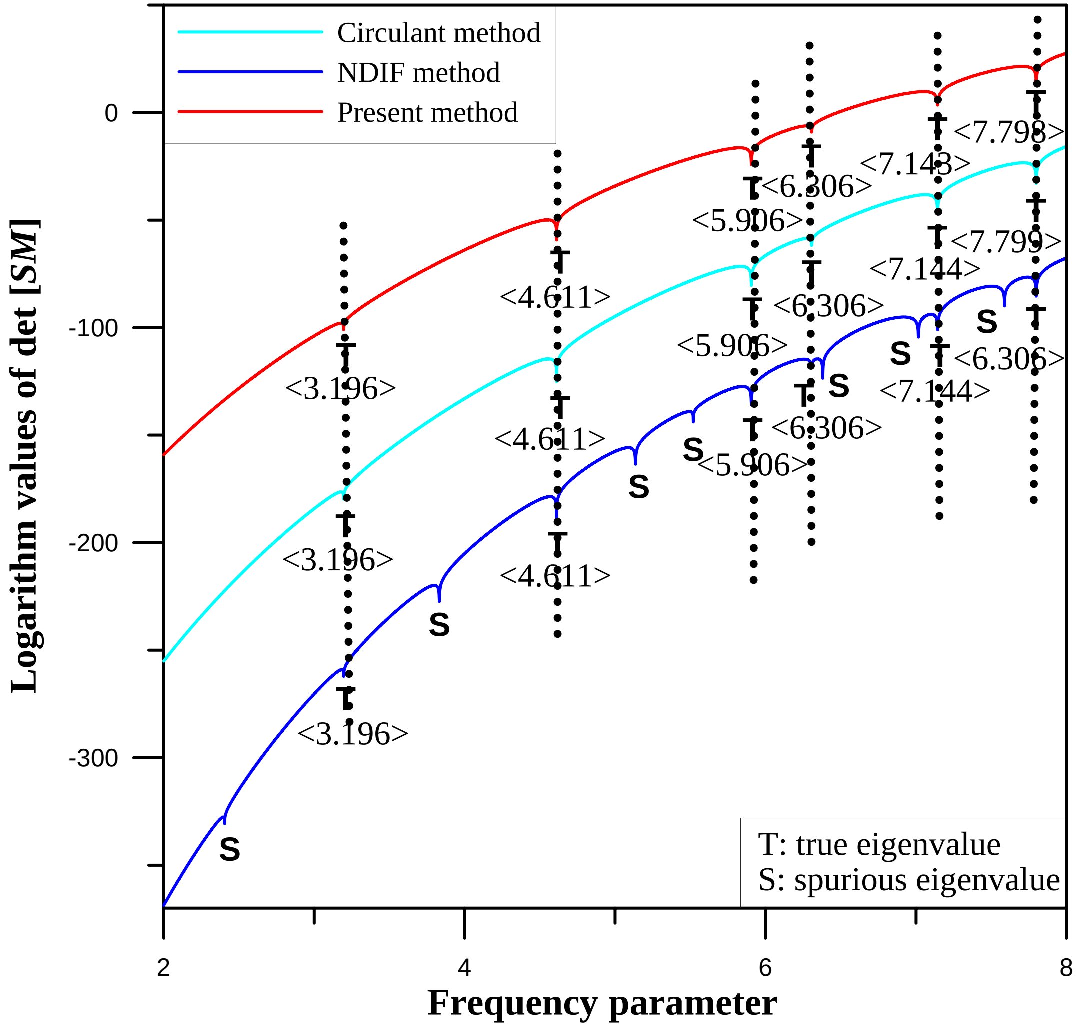
$$+\lambda\rho(-1+v)(2\lambda^2\rho^2J_{\ell+1}(\lambda\rho)I_{\ell+1}(\lambda\rho)+4\ell^2(-1+\ell)J_\ell(\lambda\rho)I_\ell(\lambda\rho)=0,$$

$$\ell=0,\pm1,\pm2,\cdots,\pm(N-1),N$$

Comparisons of the NDIF and present method

	Kang	Present method
Base	$U(s,x)=J_0(\lambda r)$ $\Theta(s,x)=I_0(\lambda r)$	$U(s,x)=\frac{1}{8\lambda^2}(J_0(\lambda r)+I_0(\lambda r))$ $\Theta(s,x)=\frac{\partial U(s,x)}{\partial n_s}$
Clamped plate	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$
	$J_\ell(\lambda r)=0$	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$
Simply-supported plate	$\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$	$\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$
	$J_\ell(\lambda r)=0$	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$
Treatment	Net approach	Dual formulation with SVD updating

Circular clamped plate using different methods



海洋大學力學聲響振動實驗室

MSVLAB HRE NTOU

Comparisons of Leissa and present method

	Leissa (Kitahara)	Present method
Clamped plate	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$
Simply-supported plate	$\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$	$\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$
Free plate J	$\frac{\lambda^2 J_\ell(\lambda\rho)+(1-\nu)[\lambda J_\ell'(\lambda\rho)-\ell^2 J_\ell(\lambda\rho)]}{\lambda^2 I_\ell(\lambda\rho)-(1-\nu)[\lambda I_\ell'(\lambda\rho)-\ell^2 I_\ell(\lambda\rho)]}=\frac{\lambda^3 I_\ell'(\lambda\rho)+(1-\nu)\ell^2[\lambda J_\ell'(\lambda\rho)-J_\ell(\lambda\rho)]}{\lambda^3 I_\ell'(\lambda\rho)-(1-\nu)\ell^2[\lambda I_\ell'(\lambda\rho)-I_\ell(\lambda\rho)]}$	$\{\ell^2(\ell^2-1)(-1+\nu)^2+\lambda^4\rho^4)(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)$ $+J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))+2\ell\lambda^2\rho^2(1-\ell)(-1+\nu)$ $(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)-J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))+\lambda\rho(-1+\nu)$ $(2\lambda^2\rho^2 J_{\ell+1}(\lambda\rho)I_{\ell+1}(\lambda\rho)+4\ell^2(-1+\ell)J_\ell(\lambda\rho)I_\ell(\lambda\rho)$ $=0$

Outlines

SVD updating terms (clamped case)

$$\left[SM^{C}\right]\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}=\begin{bmatrix}U & \Theta\\ U_{\theta} & \Theta_{\theta}\end{bmatrix}\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

$$\left[SM_{1}^{C}\right]\begin{Bmatrix}\phi'\\ \psi'\end{Bmatrix}=\begin{bmatrix}M & V\\ M_{\theta} & V_{\theta}\end{bmatrix}\begin{Bmatrix}\phi'\\ \psi'\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

SVD updating terms (clamped case)

$$[C]=\begin{bmatrix}(SM^{C})^{T}\\(SM_{1}^{C})^{T}\end{bmatrix}$$

$$=\begin{bmatrix}\Phi & 0 & 0 & 0\\0 & \Phi & 0 & 0\\0 & 0 & \Phi & 0\\0 & 0 & 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_{U} & \Sigma_{U_\theta}\\\Sigma_{\Theta} & \Sigma_{\Theta_\theta}\\\Sigma_{M} & \Sigma_{M_\theta}\\\Sigma_{V} & \Sigma_{V_\theta}\end{bmatrix}_{8N\times 4N}\begin{bmatrix}\Phi^{-1} & 0\\0 & \Phi^{-1}\end{bmatrix}$$

SVD updating terms (clamped case)

Based on the least squares

$$[C]^T[C] = \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} [D]_{4N\times 4N} \begin{bmatrix} \Phi^{-1} & 0 \\ 0 & \Phi^{-1} \end{bmatrix}$$

$$\det\left[[C]^T[C]\right] = \det[D]$$

$$= \prod_{\ell=-(N-1)}^{N} [(\lambda_\ell^{[U]}\mu_\ell^{[\Theta]} - \mu_\ell^{[U]}\lambda_\ell^{[\Theta]})^2 + (\lambda_\ell^{[U]}\mu_\ell^{[M]} - \mu_\ell^{[U]}\lambda_\ell^{[M]})^2$$

$$+ (\lambda_\ell^{[U]}\mu_\ell^{[V]} - \mu_\ell^{[U]}\lambda_\ell^{[V]})^2 + (\lambda_\ell^{[\Theta]}\mu_\ell^{[M]} - \mu_\ell^{[\Theta]}\lambda_\ell^{[M]})^2$$

$$+ (\lambda_\ell^{[\Theta]}\mu_\ell^{[V]} - \mu_\ell^{[\Theta]}\lambda_\ell^{[V]})^2 + (\lambda_\ell^{[M]}\mu_\ell^{[V]} - \mu_\ell^{[M]}\lambda_\ell^{[V]})^2]$$

SVD updating terms (clamped case)

The only possibility for zero determinant of [*D*]

$$(\lambda_\ell^{[U]}\mu_\ell^{[\Theta]}-\mu_\ell^{[U]}\lambda_\ell^{[\Theta]})=0,\quad(\lambda_\ell^{[U]}\mu_\ell^{[M]}-\mu_\ell^{[U]}\lambda_\ell^{[M]})=0,$$

$$(\lambda_\ell^{[U]}\mu_\ell^{[V]}-\mu_\ell^{[U]}\lambda_\ell^{[V]})=0,\quad(\lambda_\ell^{[\Theta]}\mu_\ell^{[M]}-\mu_\ell^{[\Theta]}\lambda_\ell^{[M]})=0,$$

$$(\lambda_\ell^{[\Theta]}\mu_\ell^{[V]}-\mu_\ell^{[\Theta]}\lambda_\ell^{[V]})=0,\quad(\lambda_\ell^{[M]}\mu_\ell^{[V]}-\mu_\ell^{[M]}\lambda_\ell^{[V]})=0.$$

at the same time for the same ℓ .

The common term is

$$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$$

True eigenequation

Outlines

Conclusions

1. Since any two combinations of the four types of potentials, six options(C_2^4) were considered.

2. Spurious eigenequation only depends on the adopted kernel function, while the true eigenequation is relevant to the specified boundary condition.

3. True eigenequation can be extract out by using SVD updating term.

MSV

1. J. T. Chen, M. H. Chang, I. L. Chung and Y. C. Cheng, 2002, Comments on eigenmode analysis of arbitrarily shaped two-dimensional cavities by the method of point matching, J. Acoust. Soc. Amer., Vol.111, No.1, pp.33-36. (SCI and EI)

2. J. T. Chen, M. H. Chang, K. H. Chen and S. R. Lin, 2002, Boundary collocation method with meshless concept for acoustic eigenanalysis of two-dimensional cavities using radial basis function, Journal of Sound and Vibration, Vol.257, No.4, pp.667-711 (SCI and EI)

3. J. T. Chen, M. H. Chang, K. H. Chen, I. L. Chen, 2002,Boundary collocation method for acoustic eigenanalysis of three-dimensional cavities using radial basis function, Computational Mechanics, Vol.29, pp.392-408. (SCI and EI)

4. J. T. Chen, S. R. Kuo, K. H. Chen and Y. C. Cheng, 2000, Comments on "vibration analysis of arbitrary shaped membranes using non-dimensional dynamic influence function“, Journal of Sound and Vibration, Vol.235, No.1, pp.156-171. (SCI and EI)

5. J. T. Chen, I. L. Chen, K. H. Chen and Y. T. Lee, 2003, Comments on “Free vibration analysis of arbitrarily shaped plates with clamped edges using wave-type function.”, Journal of Sound and Vibration, Vol.262, pp.370-378. (SCI and EI)

6. J. T. Chen, I. L. Chen, K. H. Chen, Y. T. Yeh and Y. T. Lee, 2003, A meshless method for free vibration analysis of arbitrarily shaped plates with clamped boundaries using radial basis function, Engineering Analysis with Boundary Elements, Accepted. (SCI and EI)

歡迎參觀海洋大學力學聲響振動實驗室

烘焙雞及捎來伊妹兒

http://ind.ntou.edu.tw/~msvlab/

E-mail: jtchen@mail.ntou.edu.tw

Journal of the Chinese Institute of Engineers

Call for Papers

Special Issue on Meshless Methods

If you are interested, please obser to the following schedule：

1. Abstract due： Mar. 31, 2002
2. Full paper due： Jun. 30, 2003
3. Initial acceptance/rejection： Sep. 30, 2003
4. Final, modified manuscript due：Dec. 31, 2003
5. Editing deadline： Mar. 31, 2004

Guest Editors：

Professor D. L. Young, Professor C. S. Chen, Professor A. H. –D. Cheng, Professor H. -K. Hong, Professor J. T. Chen

[第七屆邊界元素法會議](#)

[Betech 2003](#)

[ICOME 2003](#)

[Meshless work shop](#)

[國際邊界元素法電子期刊](#)

There is nothing more practical than the right theory.

Whether the theory is right or not depends on the experiment

Thanks for your kind attention