Recent development of BEM/BIEM in vibration and acoustics

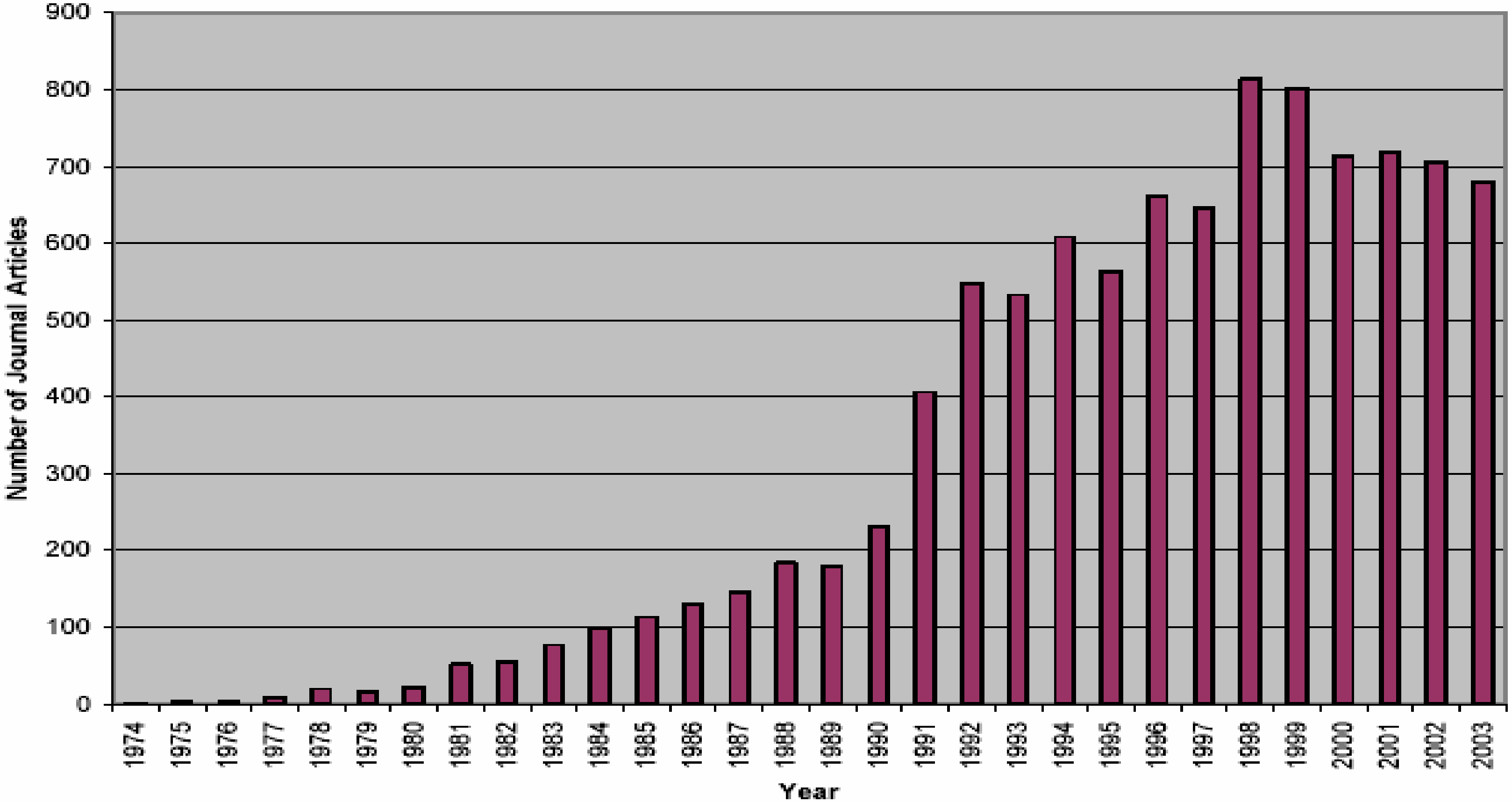
陳正宗
海洋大學 特聘教授
河海工程學系

Nov. 19, 2004, NSYSU, 14:10~16:00

Outlines

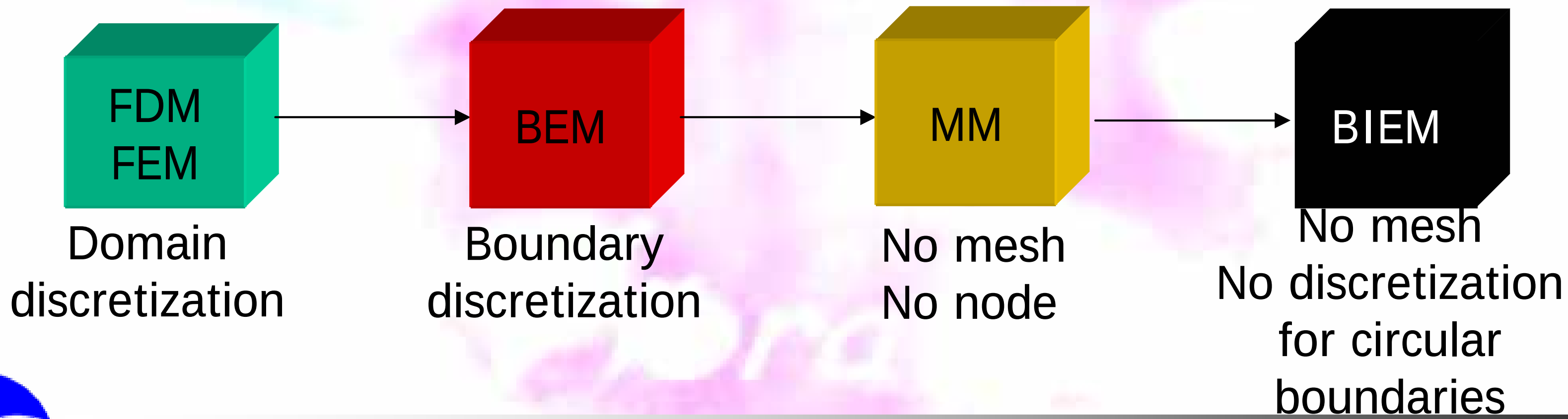
- Introduction
- Exterior acoustics - adaptive BEM
- Interior acoustics - multiply-connected eigenproblems
- Conclusions

Growth of BEM papers



Introduction

- Finite difference method (FDM)
- Finite element method (FEM)
- Boundary element method (BEM)
- Meshless method (MM)
- Boundary integral equation method (BIEM)



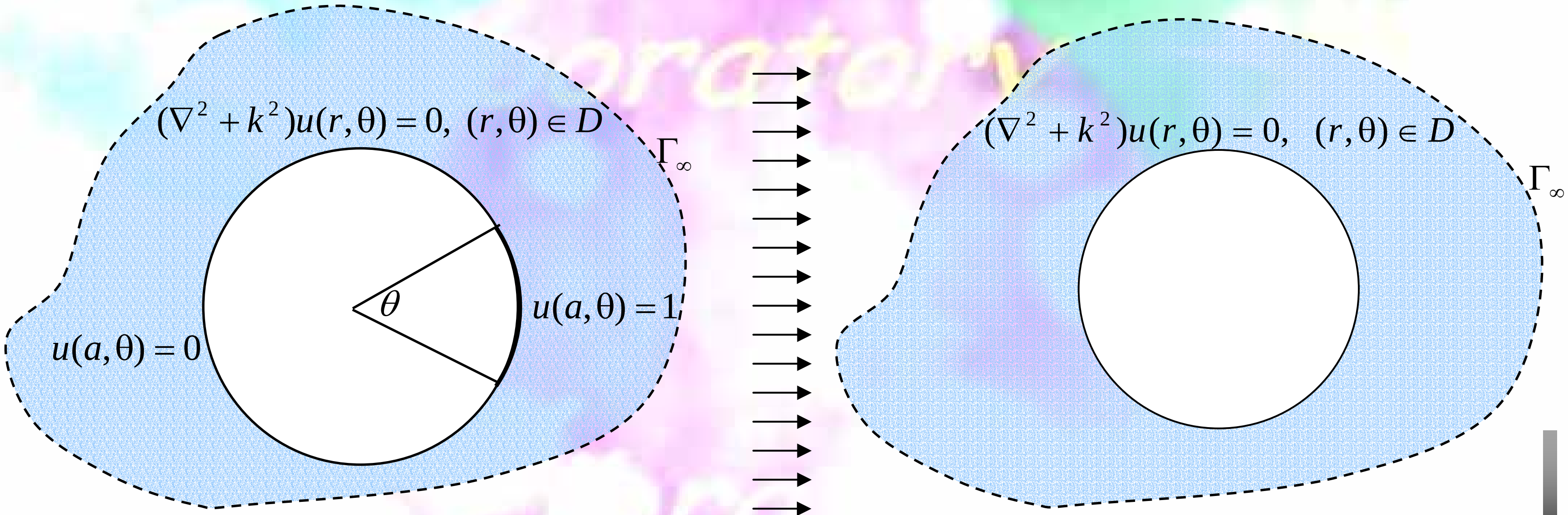
M S V

Adaptive BEM for exterior radiation and scattering problems

Problem statement

Non-uniform radiator problem

Scattering problem



Adaptive scheme

Singular formulation

$$\pi u(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - R.P.V.\int_B U(s,x)t(s)dB(s) \longrightarrow$$ **Solver**

Hypersingular formulation

$$\pi t(x) = H.P.V.\int_B M(s,x)u(s)dB(s) - C.P.V.\int_B L(s,x)t(s)dB(s) \longrightarrow$$ **Error indicator**

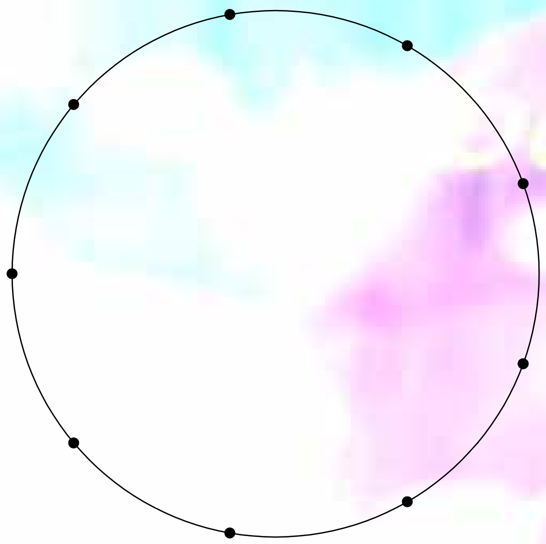
R.P.V. is Riemann Principal Value

C.P.V. is Cauchy Principal Value

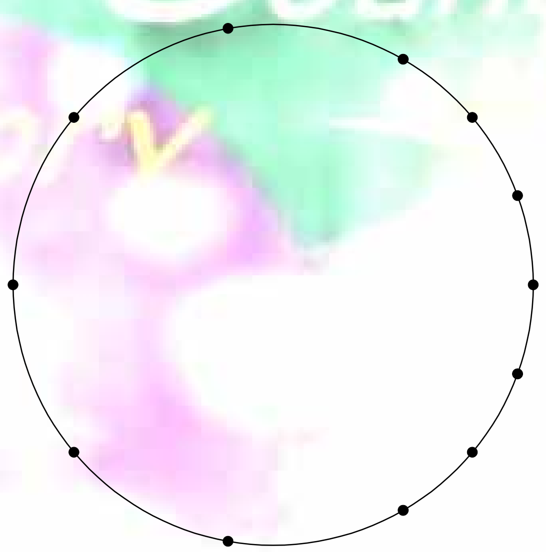
H.P.V. is Hadamard Principal Value

Adaptive mesh

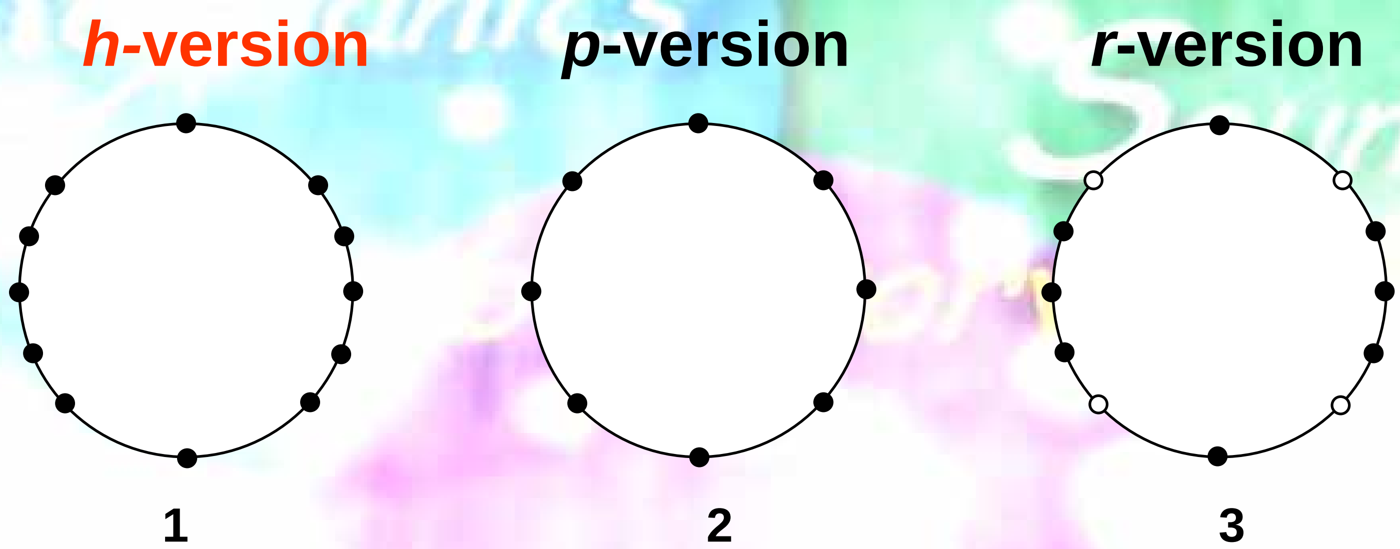
Uniform mesh



Adaptive mesh



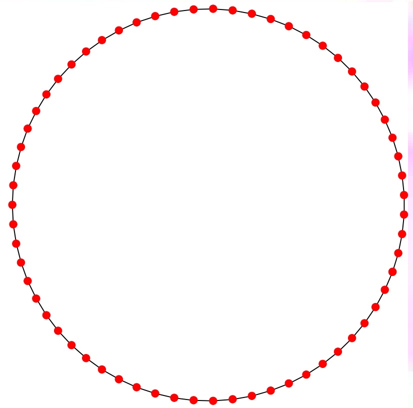
Refinement scheme



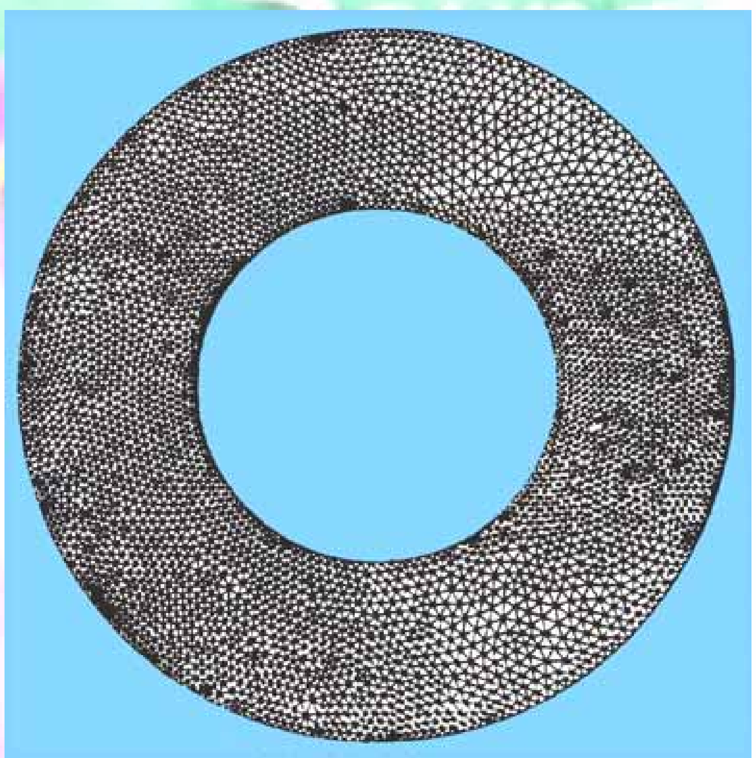
1. Element number increasing
2. Interpolation function order increasing
3. Optimum nodal collocation

Mesh

BEM



FEM(DtN)



Taiwan, NTOU

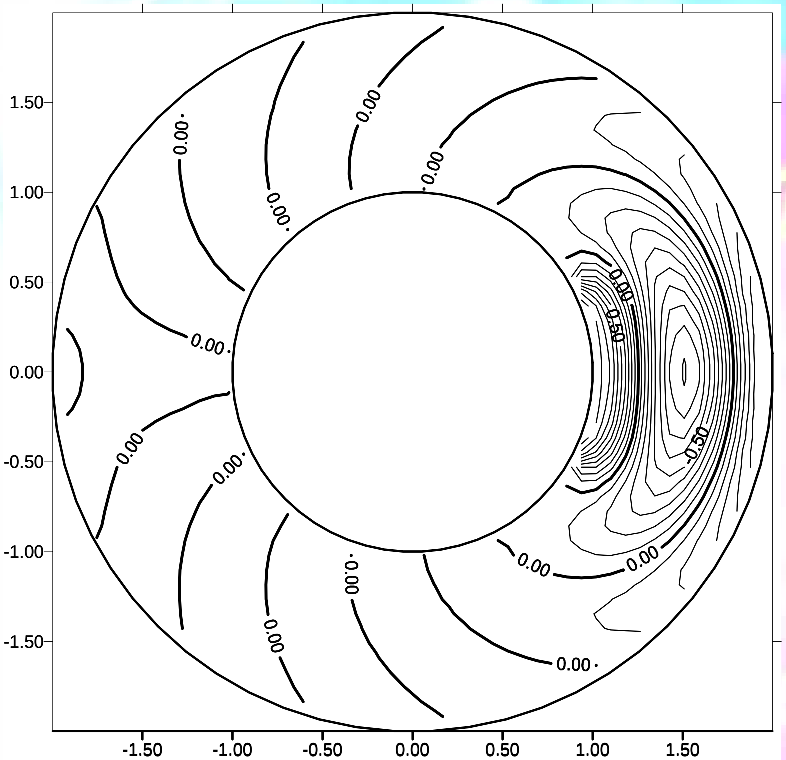
US Navy. Stanford Univ.

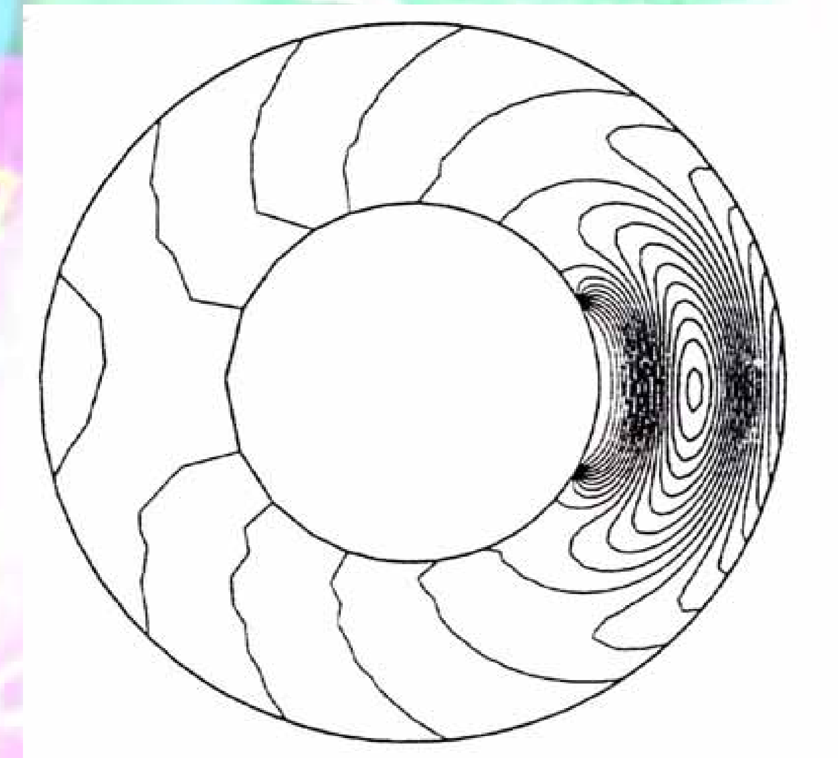
Non-uniform radiation：Dirichlet problem

Numerical solution: BEM
64 ELEMENTS

Numerical solution: FEM(DtN)
2791 ELEMENTS

$ka = 2\pi$





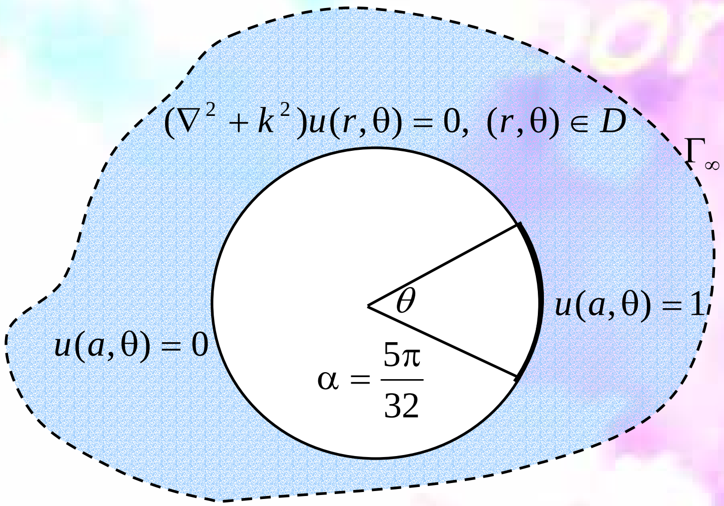
Taiwan, NTOU

US Navy. Stanford Univ.

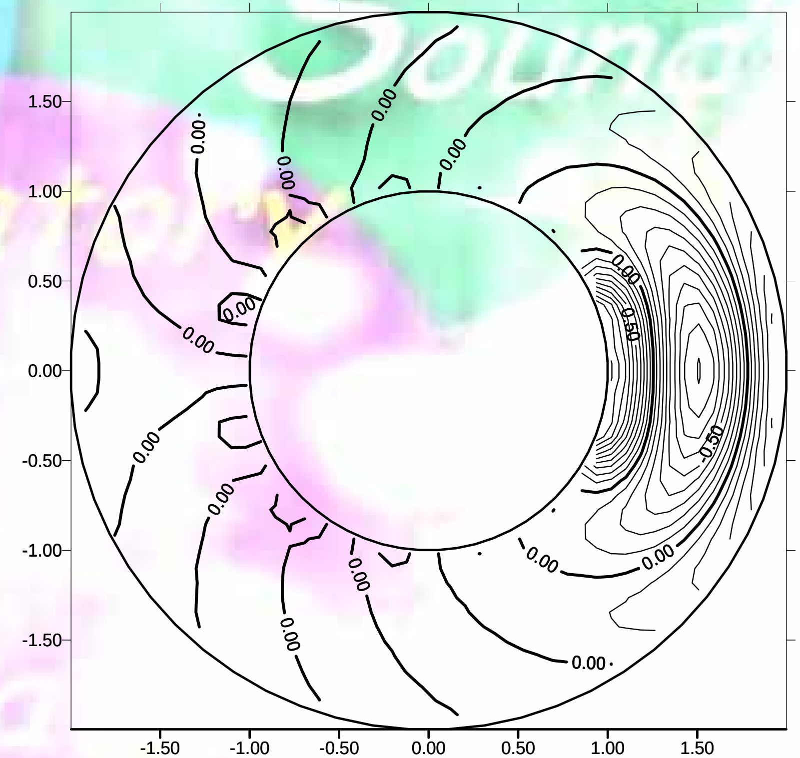
Non-uniform radiation ： Dirichlet problem

Analytical solution: n=20

$$u(r,\theta)=\frac{2}{\pi}\sum_{n=0}^{\infty}{}' \frac{\sin n\alpha}{n}\frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)}\cos n\theta$$



$ka=2\pi$



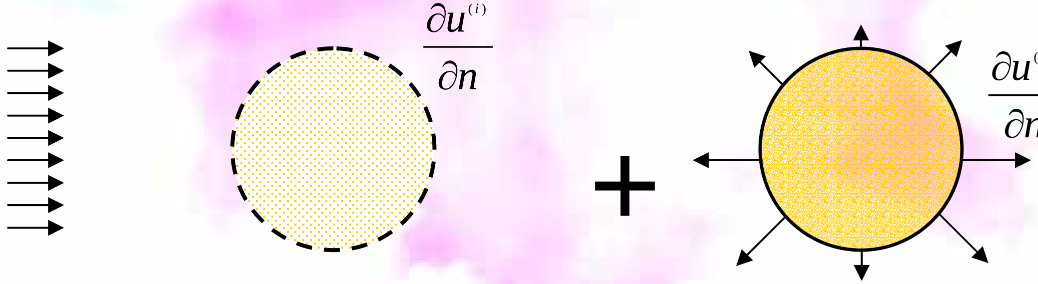
Superposition principle

$$u(r,\theta)=u^{(i)}(r,\theta)+u^{(s)}(r,\theta)$$

$$\frac{\partial u(r,\theta)}{\partial n}=0$$

$$u^{(s)}(r,\theta)$$

$$u^{i}(r,\theta)=e^{ikr\cos\theta}$$



$$\frac{\partial u^{(i)}}{\partial n}$$

$$\frac{\partial u^{(s)}}{\partial n}=-\frac{\partial u^{(i)}}{\partial n}$$

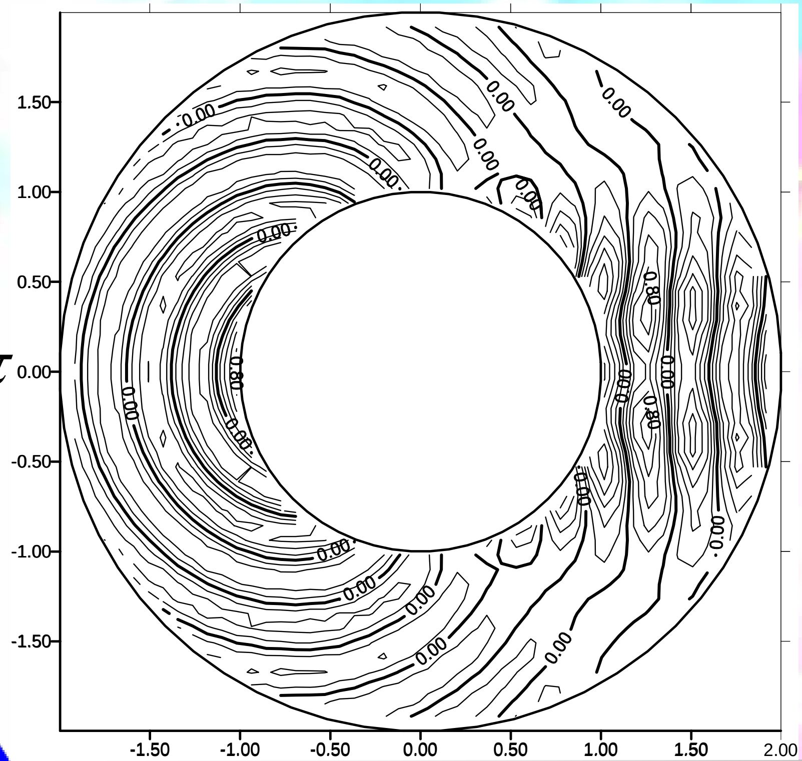
Scattering ：Neumann problem

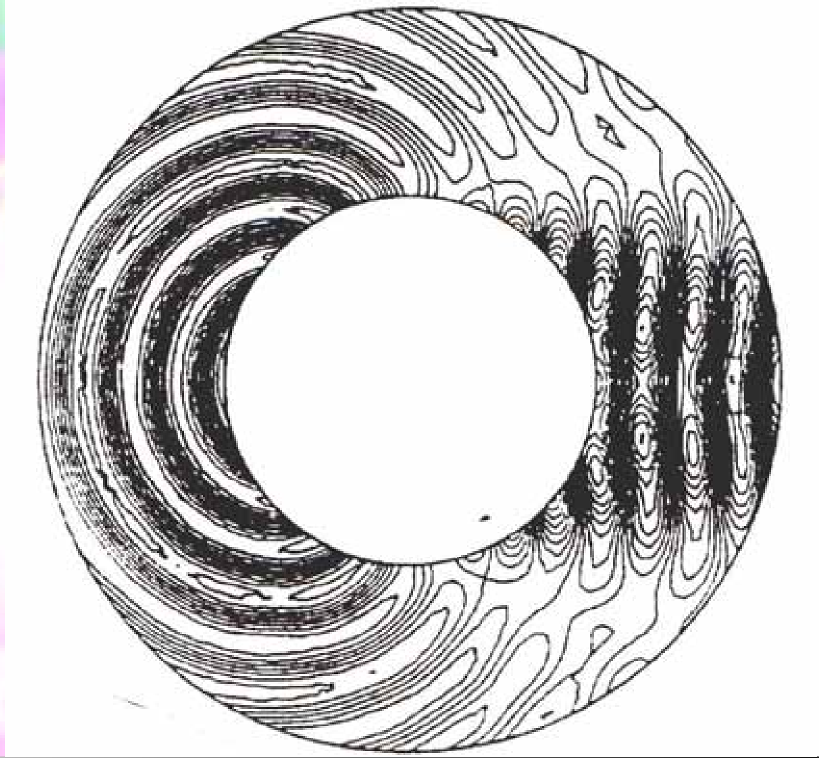
S M V

Numerical solution: BEM | **Numerical solution: FEM(DtN)**

63 ELEMENTS | **7816 ELEMENTS**

$ka = 4\pi$





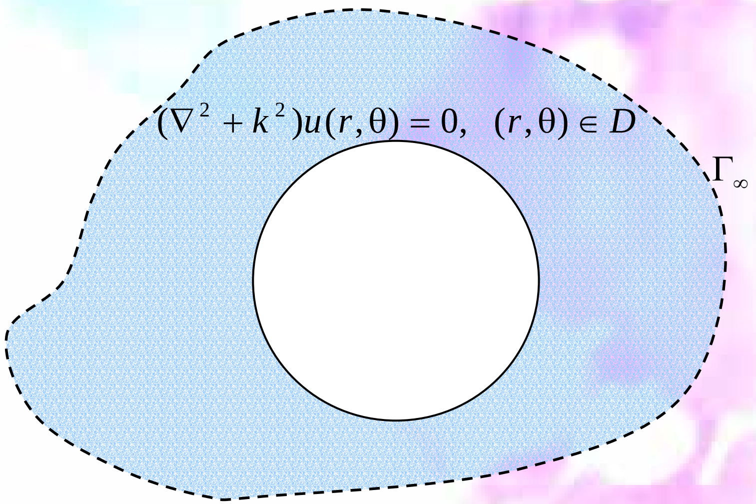
Taiwan, NTOU

US Navy. Stanford Univ.

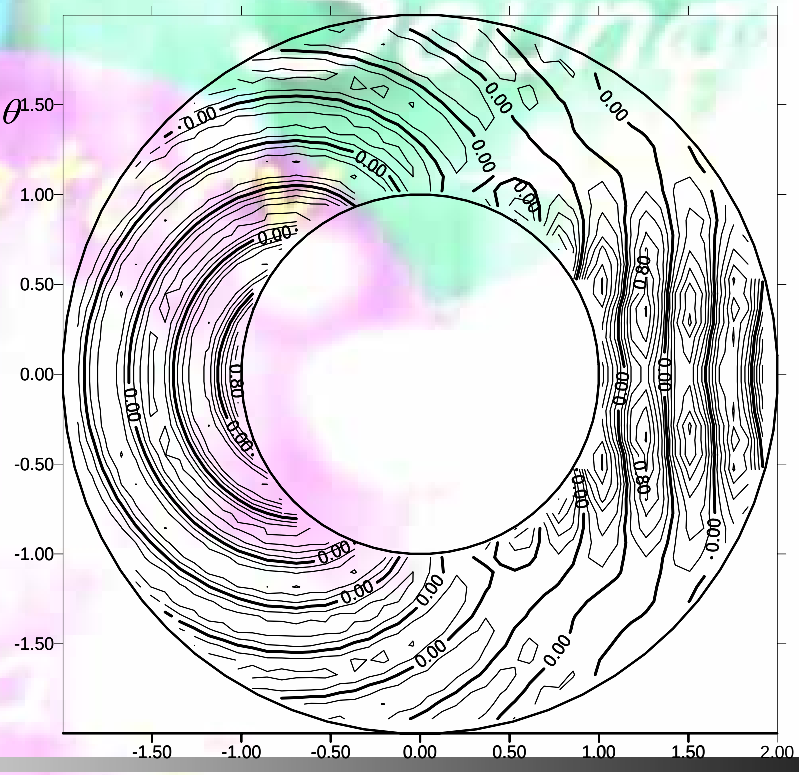
Scattering ： Neumann problem

Analytical solution: n=20

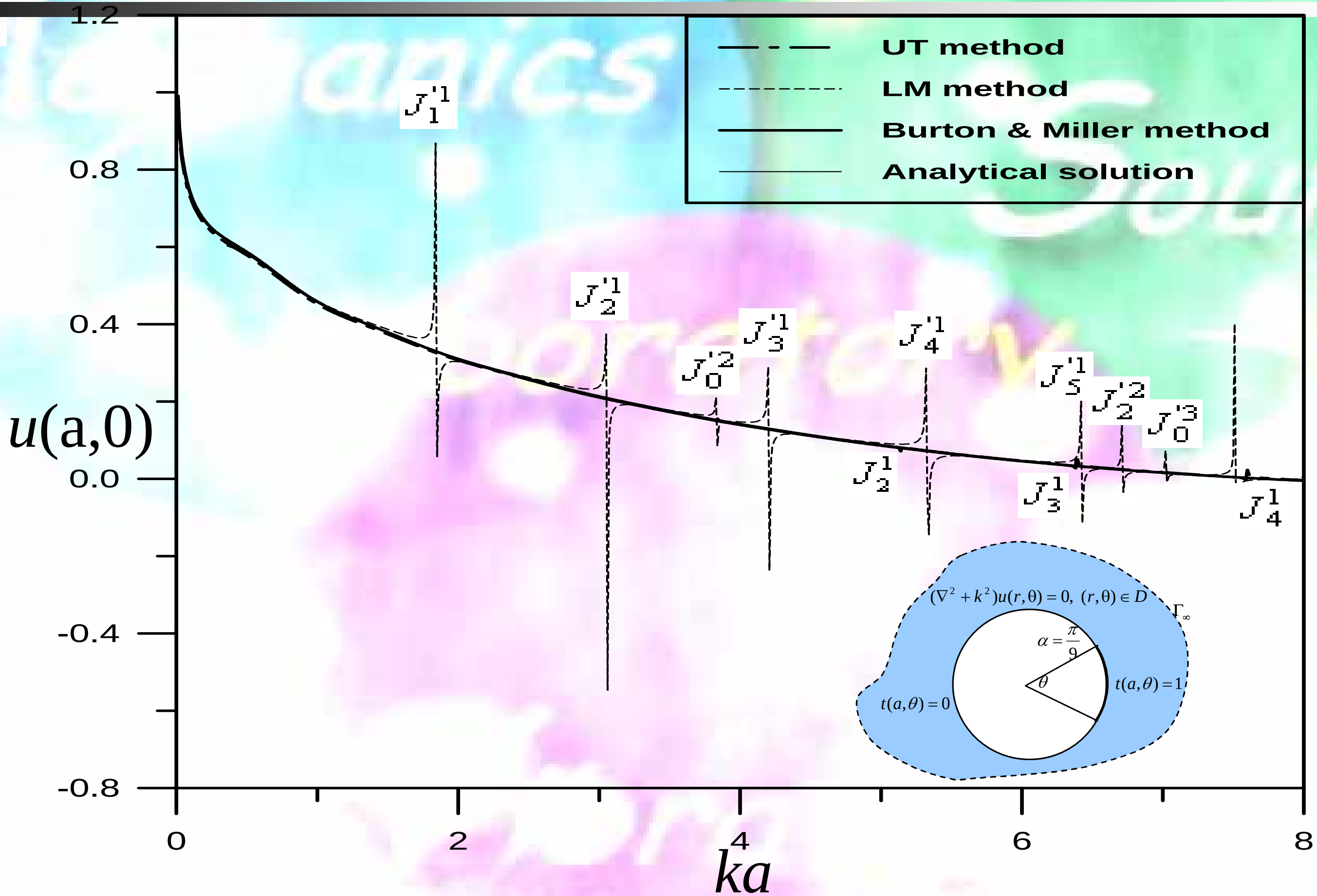
$$u(r,\theta) = -\frac{J_0'(ka)}{H_0^{(1)'}(ka)} H_0^{(1)}(kr) - 2\sum_{n=1}^{\infty} i^n \frac{J_n'(ka)}{H_n^{(1)'}(ka)} H_n^{(1)}(kr)\cos n\theta$$



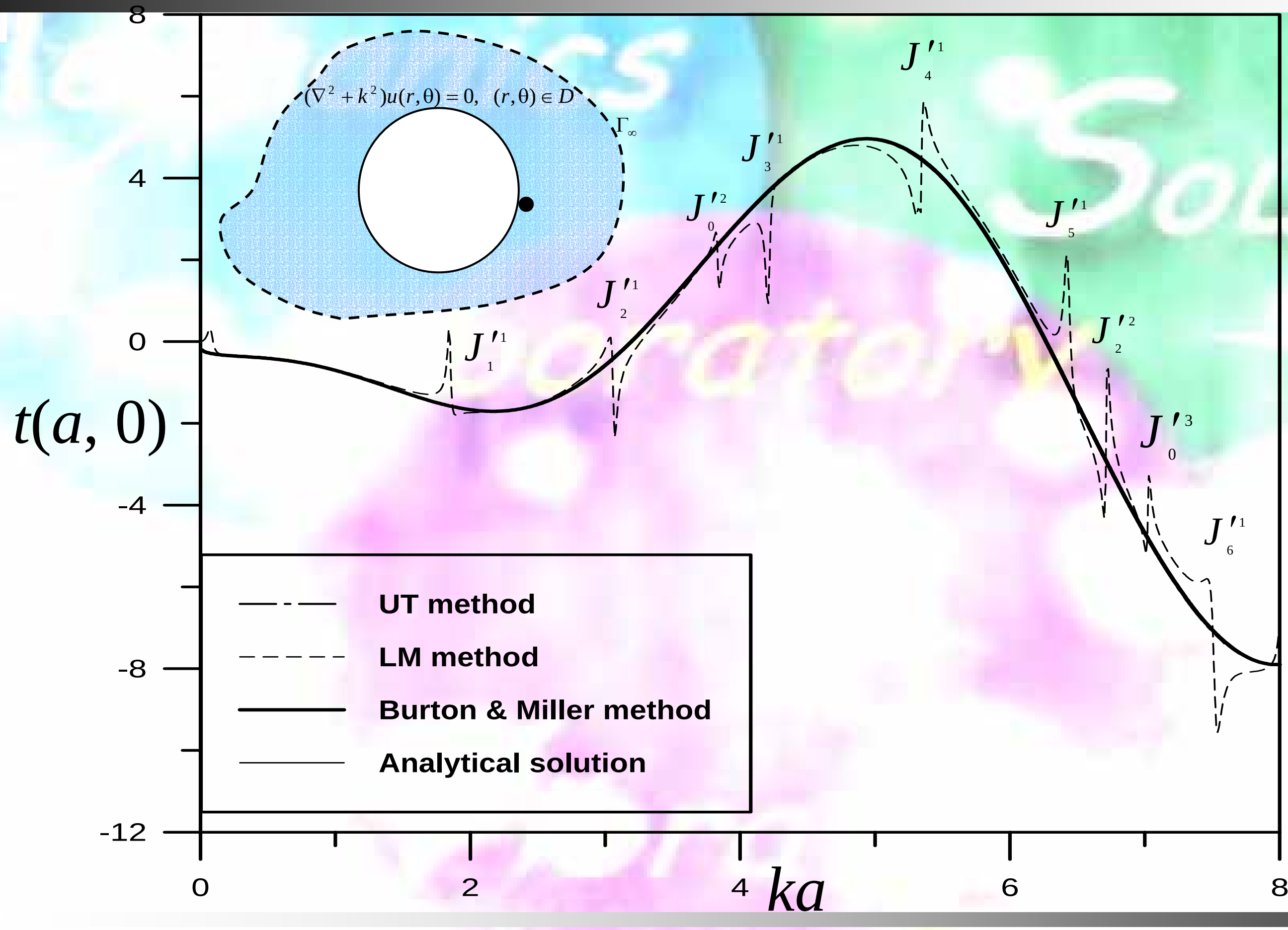
$ka = 4\pi$



Fictitious frequency： Non-uniform radiation problem



Fictitious frequency ： The scattering Dirichlet problem

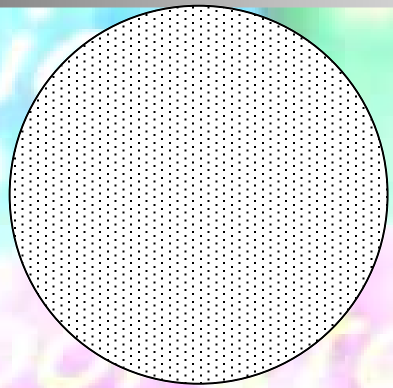


Summary

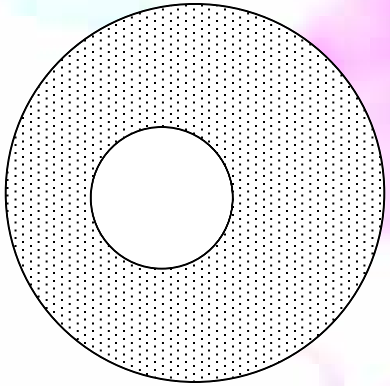
- Fictitious frequency depends on the formulation (singular or hypersingular) instead of B.C. (Dirichlet or Neumann).
- Burton & Miller method and CHIEEF method can overcome the problem of fictitious frequency.
- Fictitious frequency happens to be the true eigenvalues of the interior problem (Singular→Dirichlet, Hypersingular→Neumann).

Spurious eigenvalues for multiply-connected problems

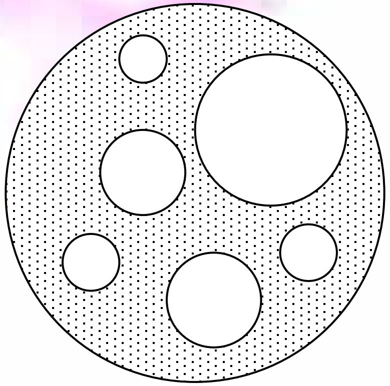
Problem domain



Simply-connected domain



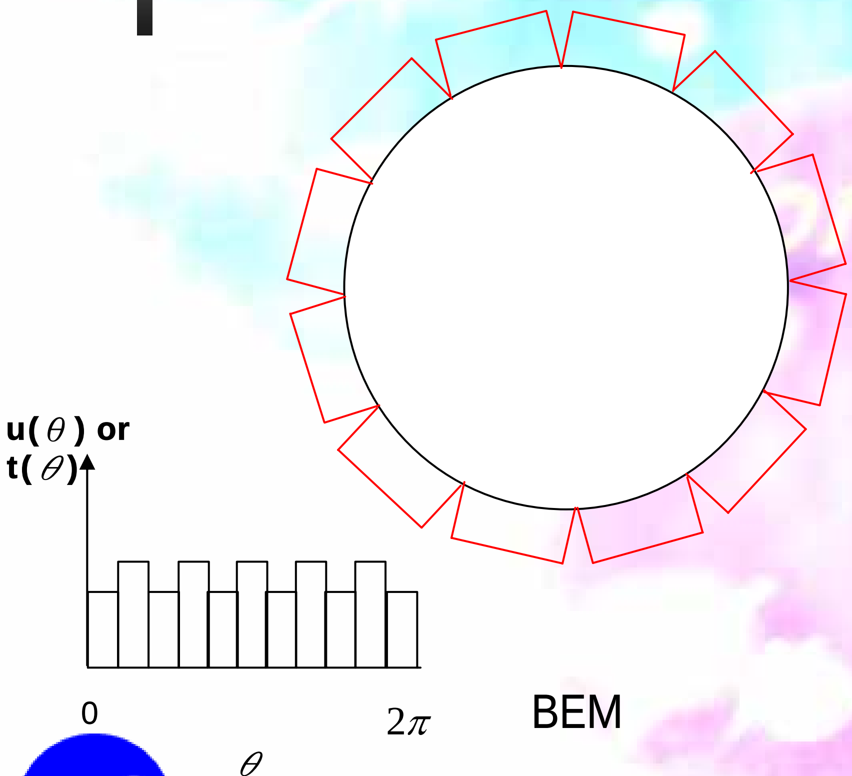
Doubly-connected domain



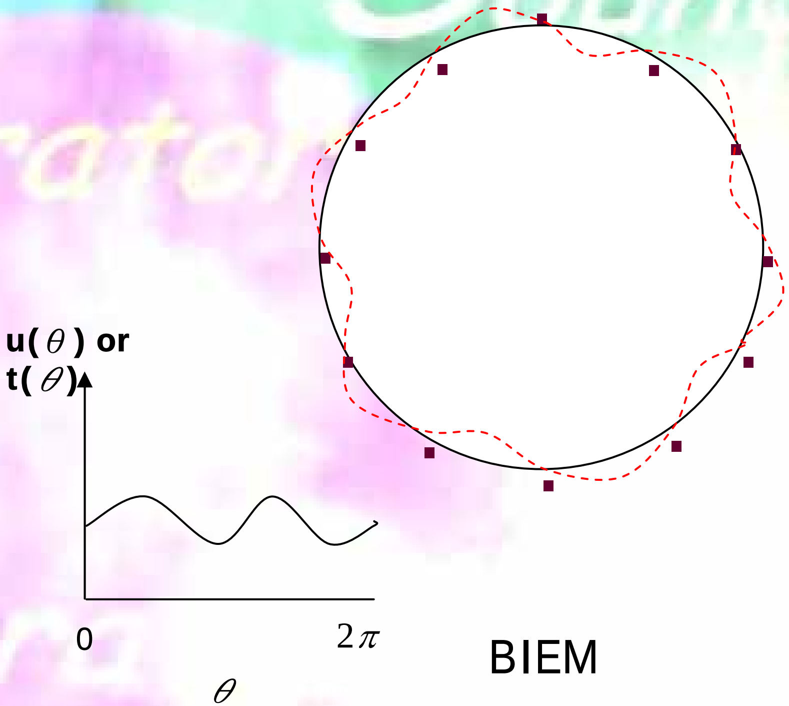
Multiply-connected domain

BEM&BIEM

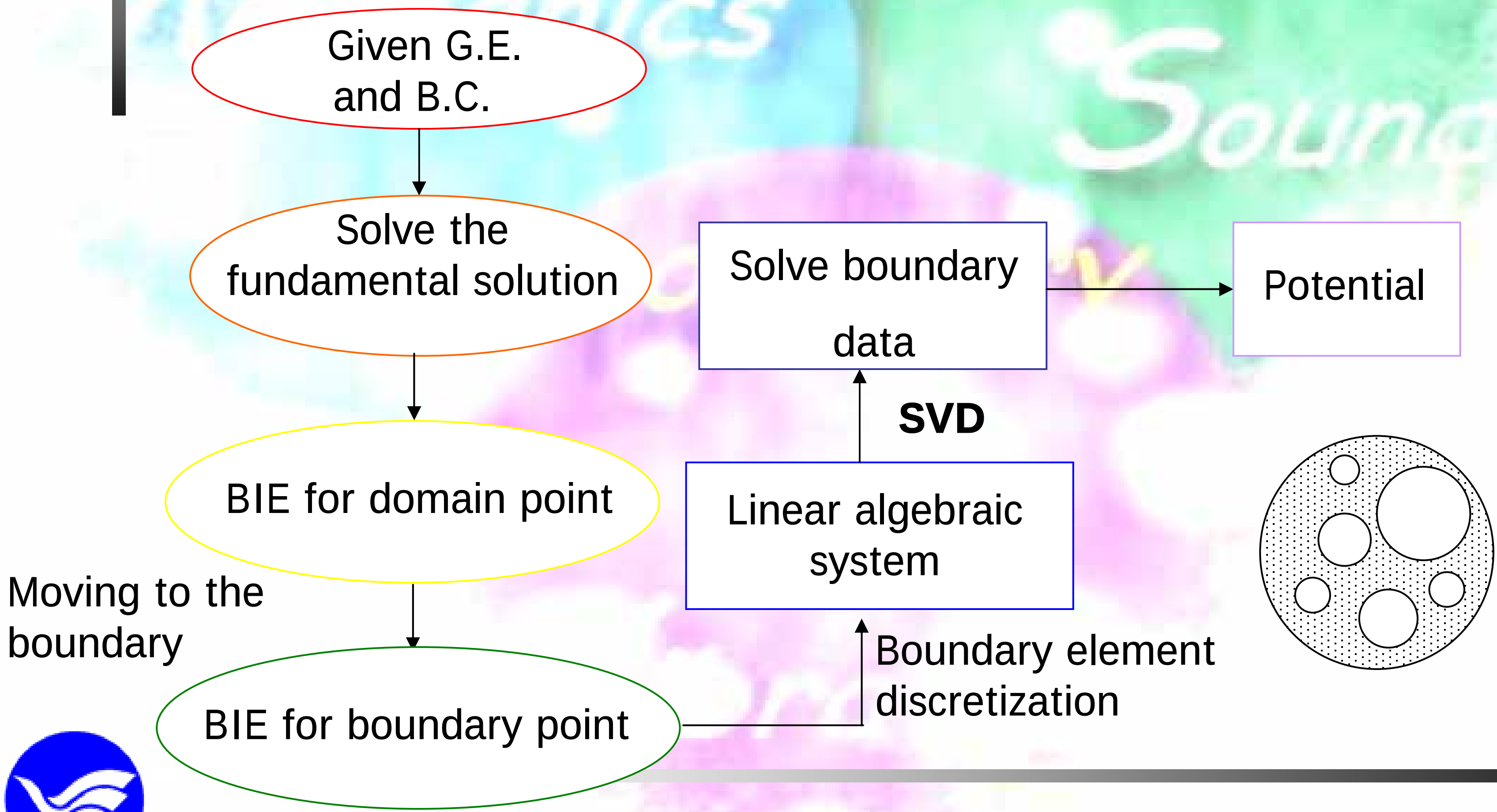
Boundary discretization



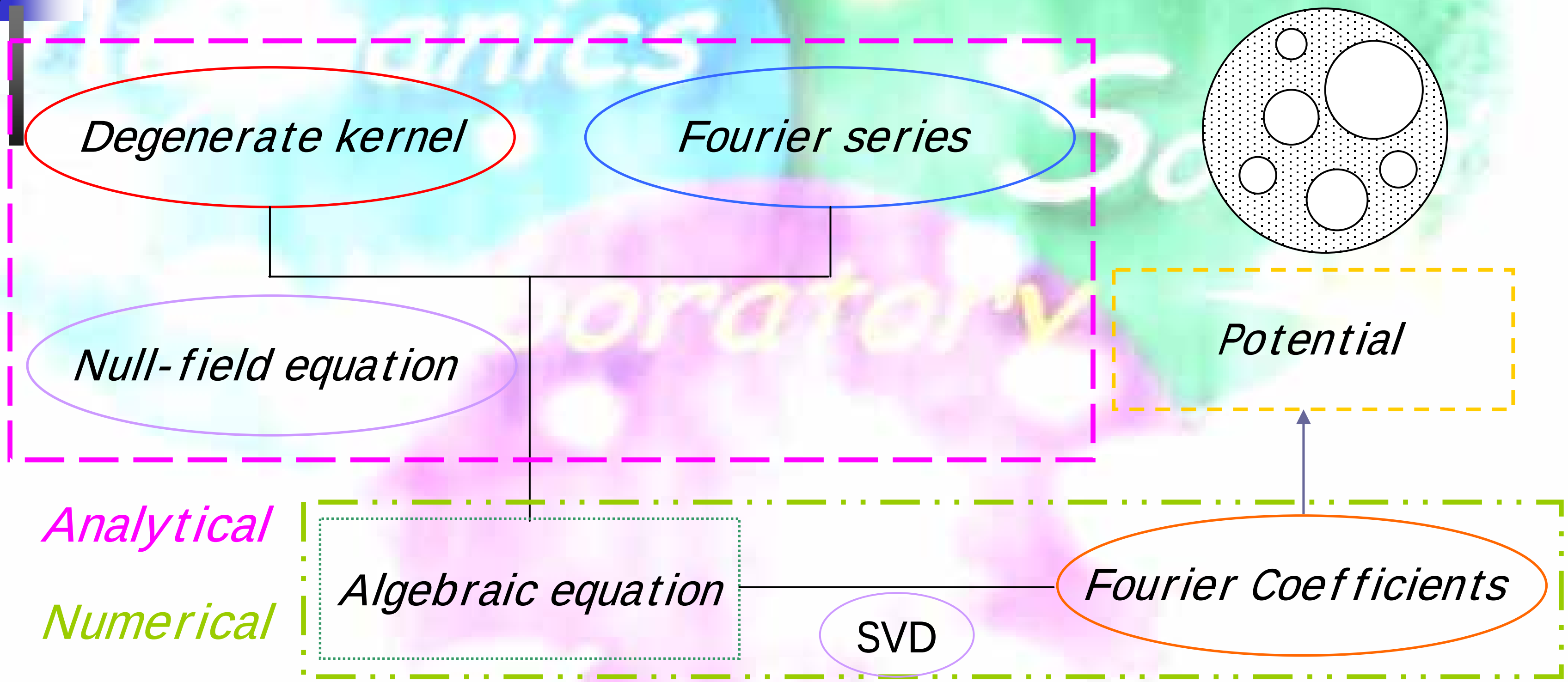
Fourier series



The flowchart to determine the eigenvalues and mode shape by BEM



The flowchart to determine the eigenvalues and mode shape by BIEM

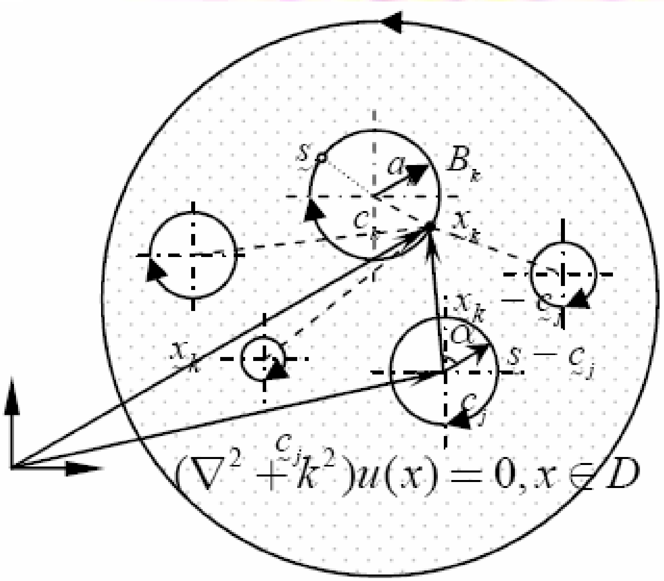


Integral Formulation

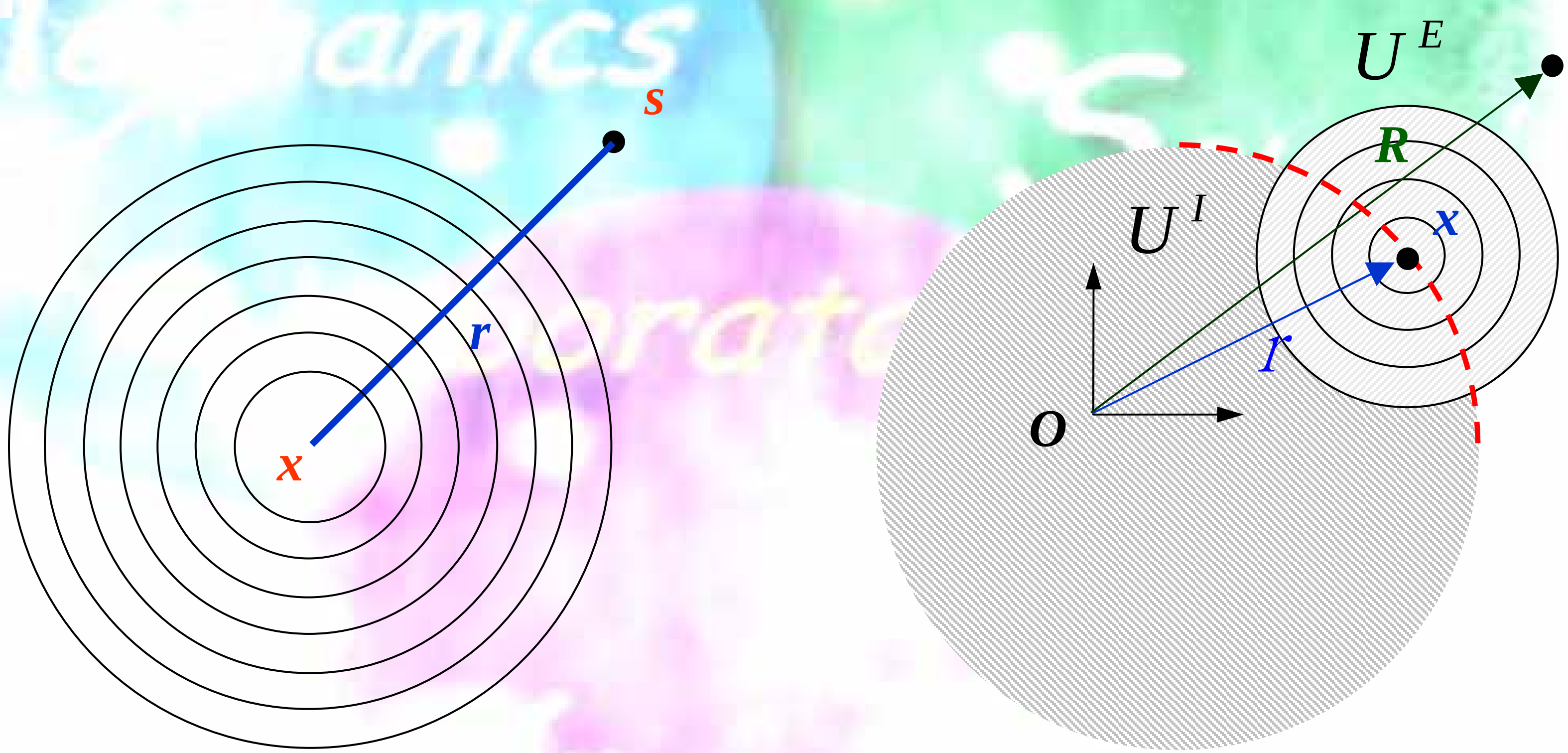
Null-field integral equations:

$$0 = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s), \quad x \in D^e.$$

$$0 = \int_B M(s,x)u(s)dB(s) - \int_B L(s,x)t(s)dB(s), \quad x \in D^e.$$



Degenerate kernels



x : source point ; s : field point

Degenerate kernels

Degenerate kernels:

$$U(s,x)=\begin{cases} U^I(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\sum_{m=0}^{\infty}\varepsilon_m J_m(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)H_m^{(1)}(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)\cos(m\alpha), |\underset{\sim}{s}-\underset{\sim}{c}_j|>|\underset{\sim}{x}-\underset{\sim}{c}_j| \\ U^E(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\sum_{m=0}^{\infty}\varepsilon_m H_m^{(1)}(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)J_m(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)\cos(m\alpha), |\underset{\sim}{x}-\underset{\sim}{c}_j|>|\underset{\sim}{s}-\underset{\sim}{c}_j| \end{cases}$$

$$\varepsilon_m=\begin{Bmatrix} 1, & m=0, \\ 2, & m\neq 0, \end{Bmatrix}.$$

$$T(s,x)=\begin{cases} T^I(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\sum_{m=0}^{\infty}\varepsilon_m J_m(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)\{\dfrac{\partial H_m^{(1)}(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)}{\partial R_j}\}\cos(m\alpha), \quad |\underset{\sim}{s}-\underset{\sim}{c}_j|>|\underset{\sim}{x}-\underset{\sim}{c}_j| \\ T^E(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\sum_{m=0}^{\infty}\varepsilon_m H_m^{(1)}(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)\{\dfrac{\partial J_m(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)}{\partial R_j}\}\cos(m\alpha), \quad |\underset{\sim}{x}-\underset{\sim}{c}_j|>|\underset{\sim}{s}-\underset{\sim}{c}_j| \end{cases}$$

Degenerate kernels

$$L(s,x)=\begin{cases} L^I(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty}\varepsilon_m H_m^{(1)}(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)\{\dfrac{\partial J_m(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)}{\partial\rho_j}\cos(m\alpha)\cos(\phi_c-\phi_j) \\ \qquad +\dfrac{1}{\rho_j}J_m(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)\dfrac{\partial\cos(m\alpha)}{\partial\phi_j}\cos(\dfrac{\pi}{2}-\phi_c+\phi_j)\}, \quad |\underset{\sim}{s}-\underset{\sim}{c}_j|>|\underset{\sim}{x}-\underset{\sim}{c}_j| \\ L^E(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty}\varepsilon_m J_m(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)\{\dfrac{\partial H_m^{(1)}(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)}{\partial\rho_j}\cos(m\alpha)\cos(\phi_c-\phi_j) \\ \qquad +\dfrac{1}{\rho_j}H_m^{(1)}(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)\dfrac{\partial\cos(m\alpha)}{\partial\phi_j}\cos(\dfrac{\pi}{2}-\phi_c+\phi_j)\}, \quad |\underset{\sim}{x}-\underset{\sim}{c}_j|>|\underset{\sim}{s}-\underset{\sim}{c}_j| \end{cases}$$

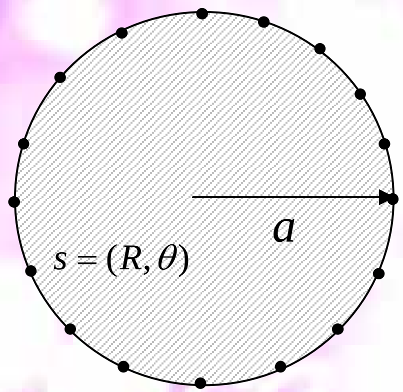
$$M(s,x)=\begin{cases} M^I(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty}\varepsilon_m \dfrac{\partial H_m^{(1)}(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)}{\partial R_j}\{\dfrac{\partial J_m(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)}{\partial\rho_j}\cos(m\alpha)\cos(\phi_c-\phi_j) \\ \qquad +\dfrac{1}{\rho_j}J_m(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)\dfrac{\partial\cos(m\alpha)}{\partial\phi_j}\cos(\dfrac{\pi}{2}-\phi_c+\phi_j)\}, \quad |\underset{\sim}{s}-\underset{\sim}{c}_j|>|\underset{\sim}{x}-\underset{\sim}{c}_j| \\ M^E(\underset{\sim}{s},\underset{\sim}{x})=\dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty}\varepsilon_m \dfrac{\partial J_m(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)}{\partial R_j}\{\dfrac{\partial H_m^{(1)}(k|\underset{\sim}{x}-\underset{\sim}{c}_j|)}{\partial\rho_j}\cos(m\alpha)\cos(\phi_c-\phi_j) \\ \qquad +\dfrac{1}{\rho_j}H_m^{(1)}(k|\underset{\sim}{s}-\underset{\sim}{c}_j|)\dfrac{\partial\cos(m\alpha)}{\partial\phi_j}\cos(\dfrac{\pi}{2}-\phi_c+\phi_j)\}, |\underset{\sim}{x}-\underset{\sim}{c}_j|>|\underset{\sim}{s}-\underset{\sim}{c}_j| \end{cases}$$

Fourier series for boundary densities

Fourier series:

$$u(\tilde{s}) = a_{0j} + \sum_{n=1}^{\infty} (a_{nj}\cos n\theta_j + b_{nj}\sin n\theta_j),\ \tilde{s} \in B_j$$

$$t(\tilde{s}) = p_{0j} + \sum_{n=1}^{\infty} (p_{nj}\cos n\theta_j + q_{nj}\sin n\theta_j),\ \tilde{s} \in B_j$$

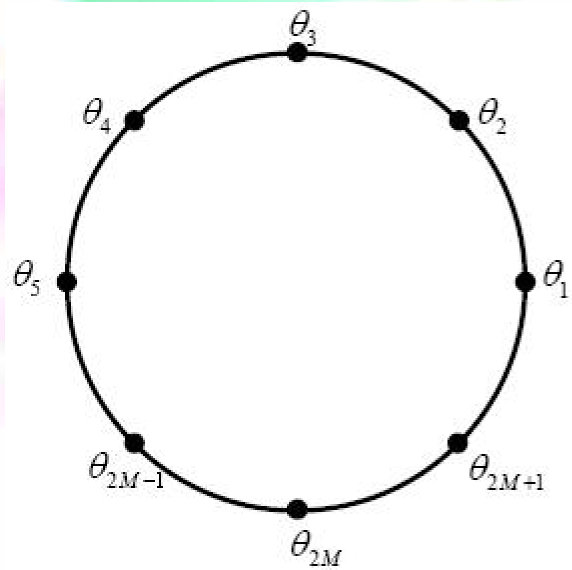


Collocation points

By choosing M terms of Fourier series, we select 2M+1 collocation points on the circle.

$$u(x) = a_0 + \sum_{n=1}^{M} (a_n \cos n\phi + b_n \sin n\phi)$$

2M+1 terms

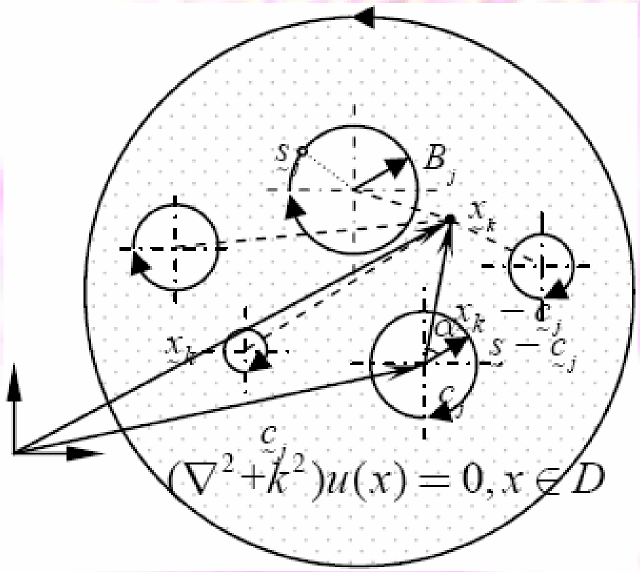


Integral representation

Integral equation formulation:

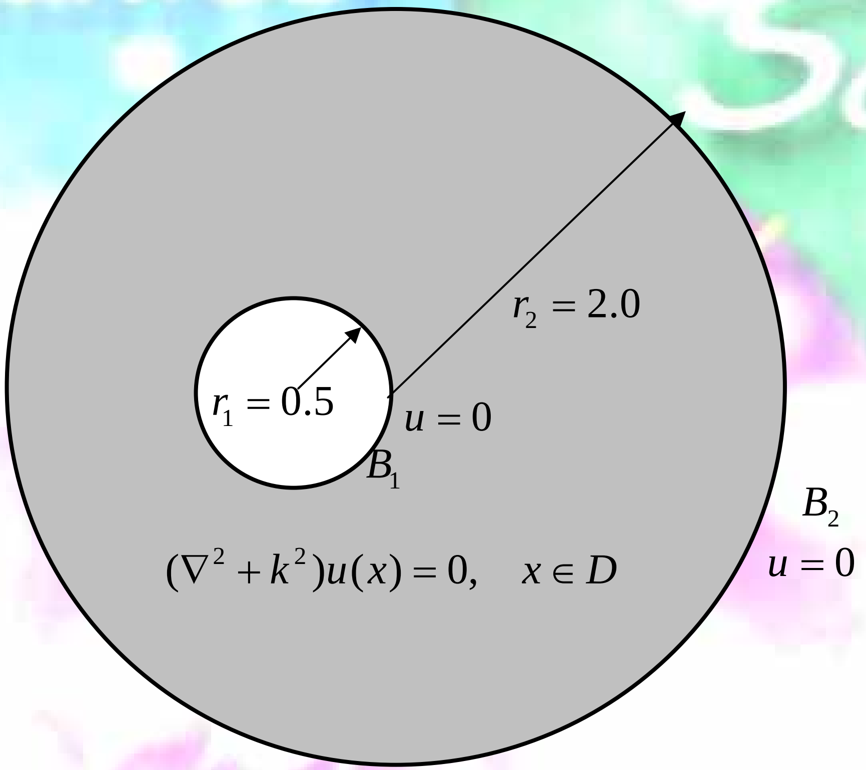
$$2\pi u(\mathbf{x}) = \int_B T(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s}),\ \mathbf{x} \in D$$

$$2\pi t(\mathbf{x}) = \int_B M(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B L(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s}),\ \mathbf{x} \in D$$

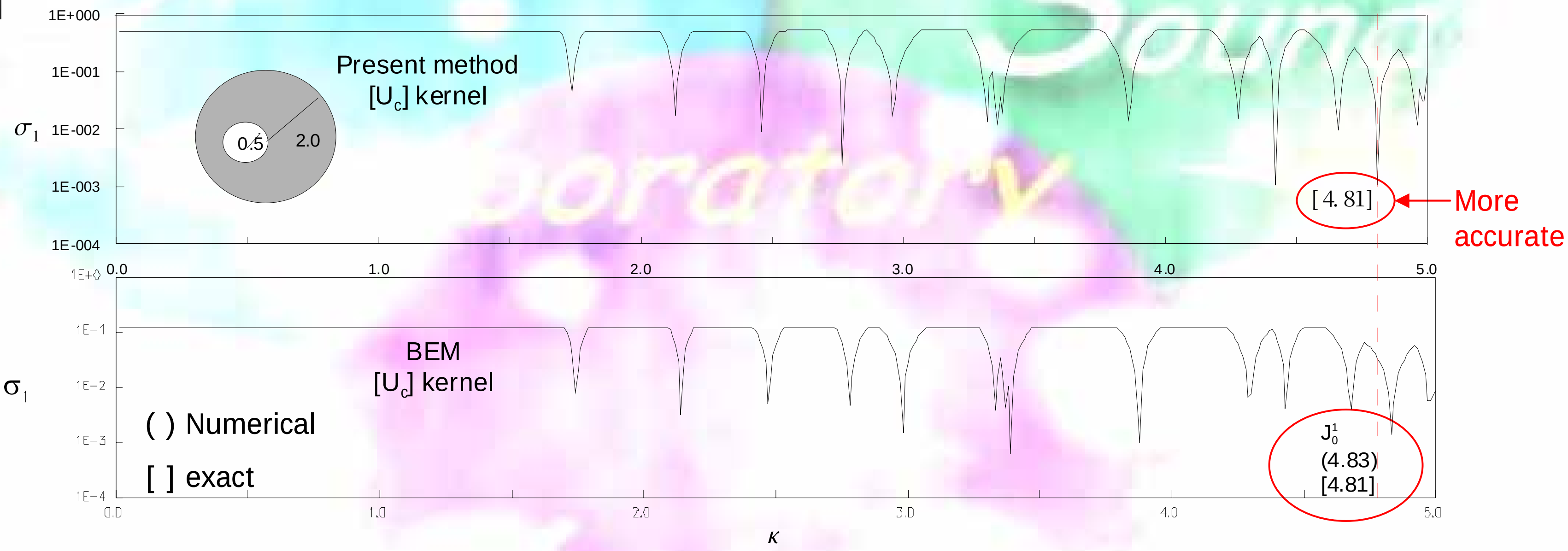


Numerical examples

Example 1



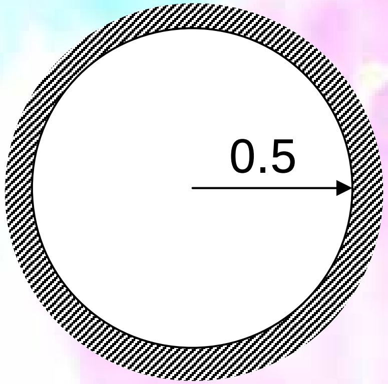
The eigenfrequenies by using singular equation



Contaminated by spurious eigenvalues

Relation of spurious eigenvalue and true eigenvalue

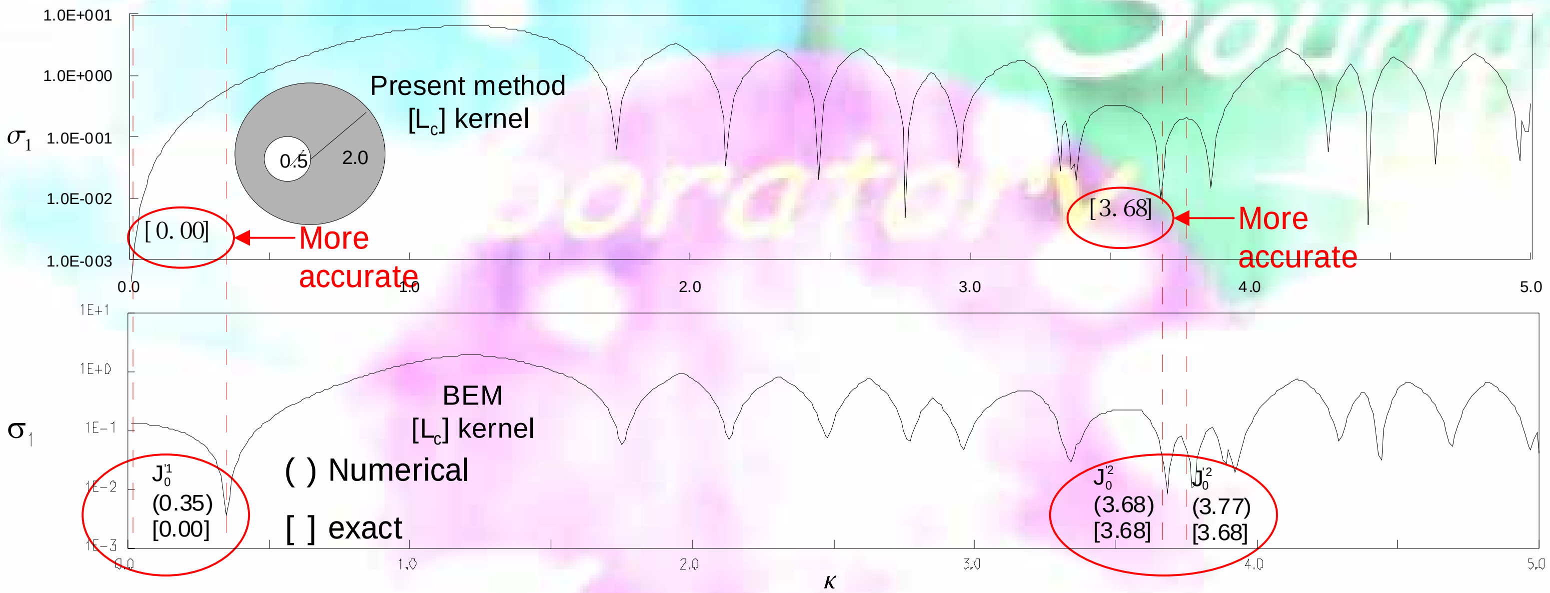
$$J_n(k \times 0.5) = 0 \quad \Rightarrow \quad k = 4.81$$



True

Spurious eigenvalue using singular formulation happens to be the true eigenvalue of the associated interior Dirichlet problem.

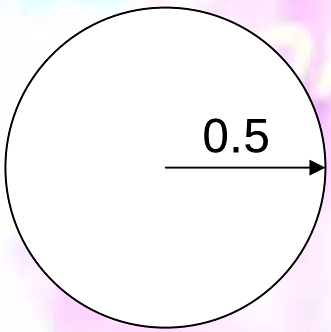
The eigenfrequenies by using hyper-singular equation



Contaminated by spurious eigenvalues

Relation of spurious eigenvalue and true eigenvalue

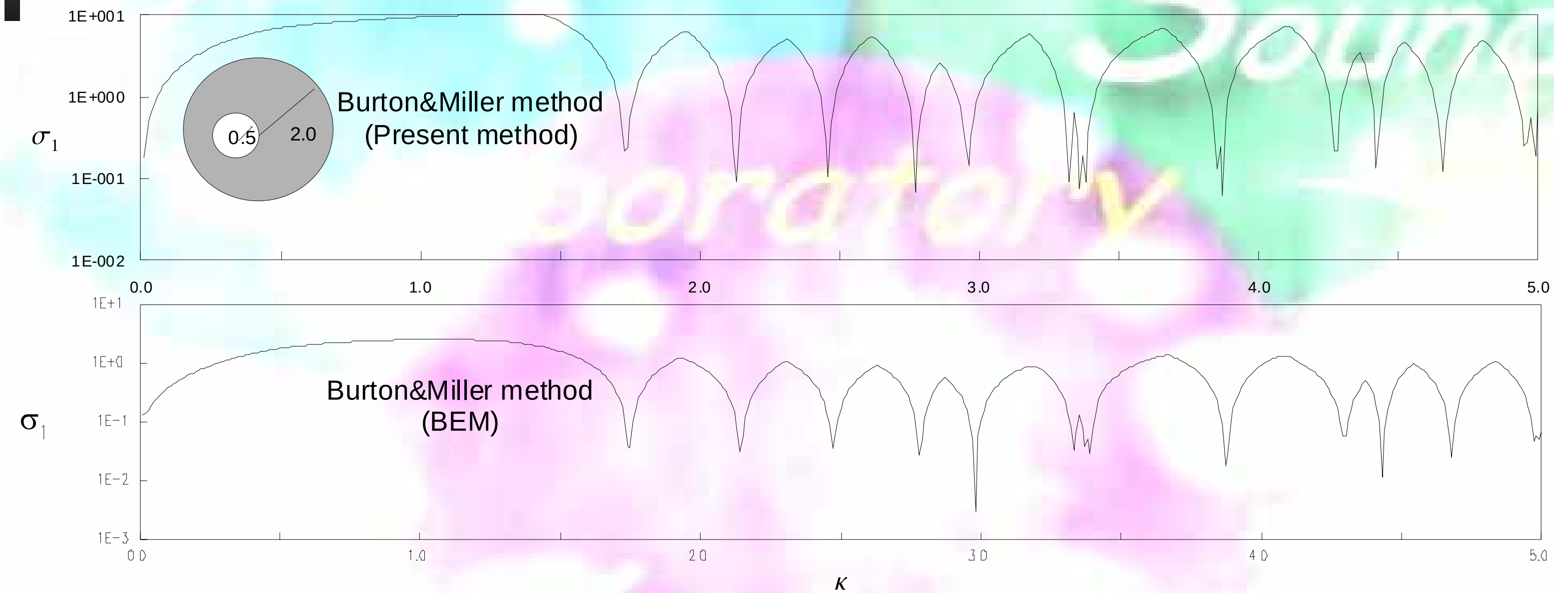
$$J_n'(k \times 0.5) = 0 \quad \Rightarrow \quad k = 0, 3.68$$



True

Spurious eigenvalue using hypersingular formulation happens to be the true eigenvalue of the associated interior Neumann problem.

The spurious eigenvalues are filtered by Burton&Miller method

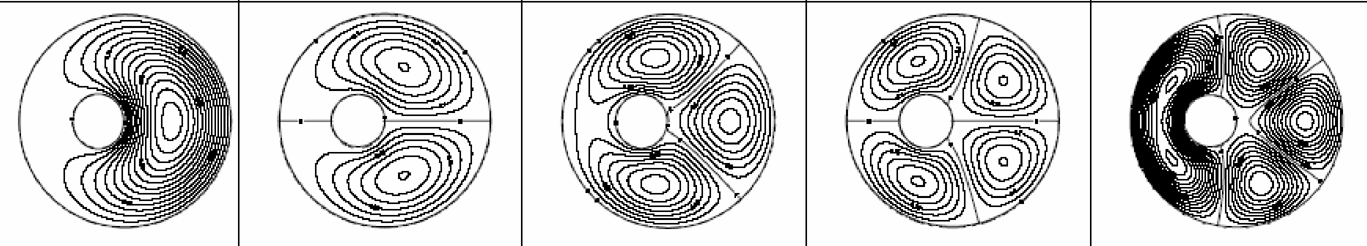
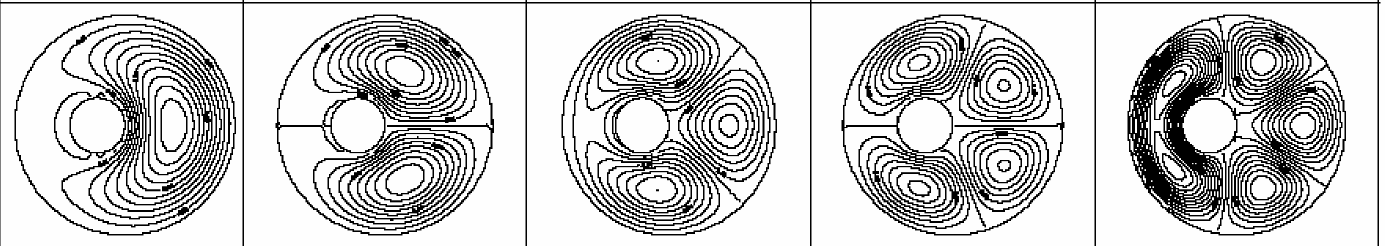
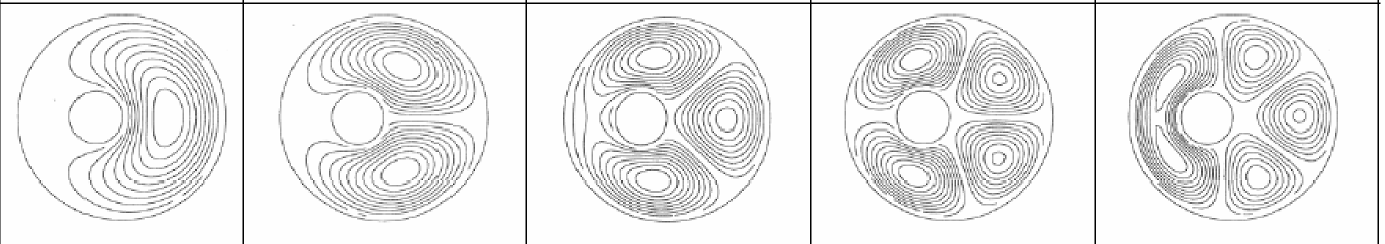


Only true eigenvalues appear

The former five eigenvalues of Helmholtz eigenproblem with an eccentric domain

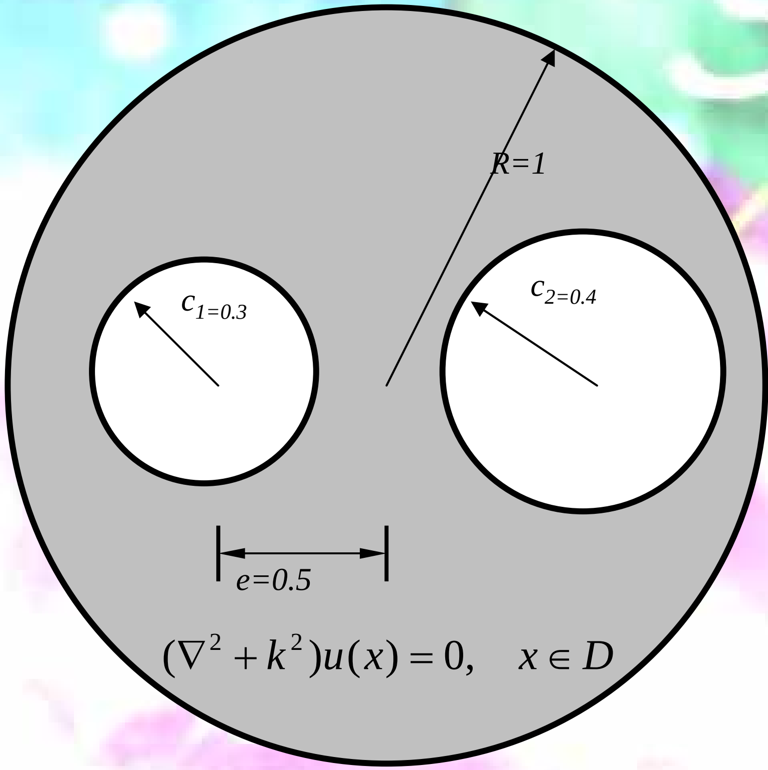
	1	2	3	4	5
FEM [Chen *et. al.*]	**1.73**	**2.13**	**2.45**	**2.76**	**2.95**
BEM [Chen and Zhou]	**1.75**	**2.14**	**2.47**	**2.78**	**2.97**
BEM [Chen *et. al.*]	**1.74**	**2.14**	**2.47**	**2.78**	**2.98**
Present method	**1.74**	**2.14**	**2.46**	**2.78**	**2.96**

The former five eigenmodes for eccentric case using present method, FEM and BEM

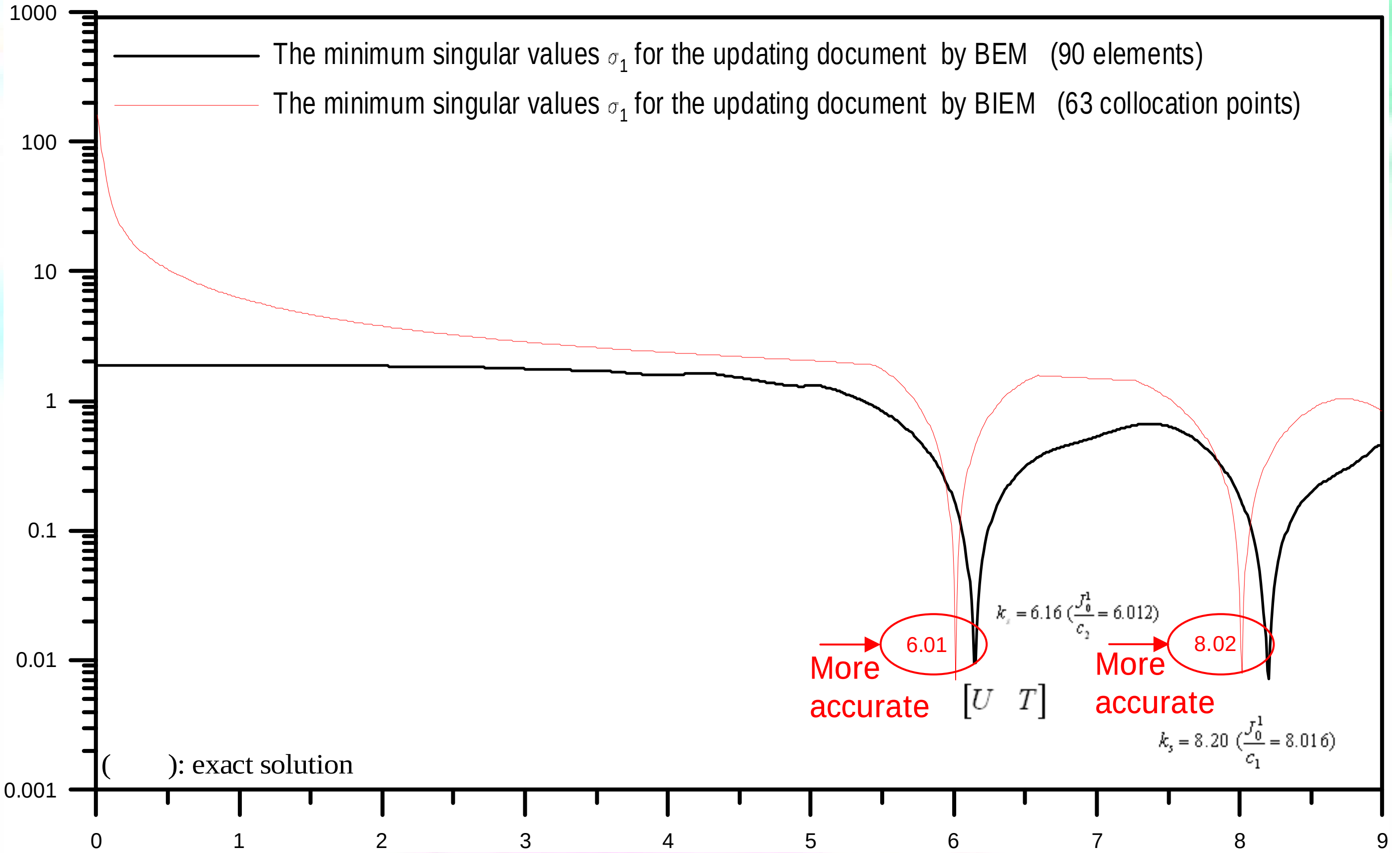
Method \ Mode	1	2	3	4	5
Present method					
	$k = 1.74$	$k = 2.14$	$k = 2.46$	$k = 2.78$	$k = 2.94$
BEM					
	$k = 1.74$	$k = 2.14$	$k = 2.47$	$k = 2.78$	$k = 2.97$
FEM					
	$k = 1.74$	$k = 2.13$	$k = 2.45$	$k = 2.76$	$k = 2.95$

Numerical examples

Example 2



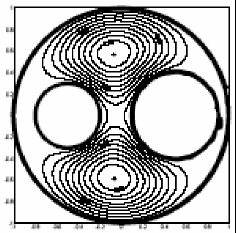
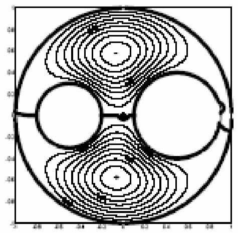
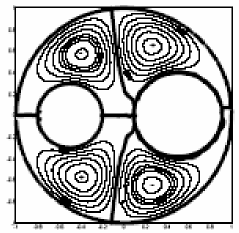
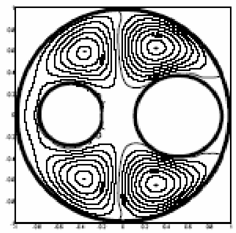
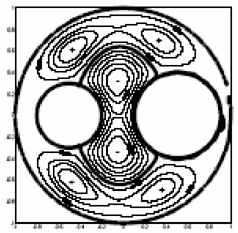
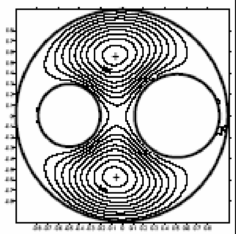
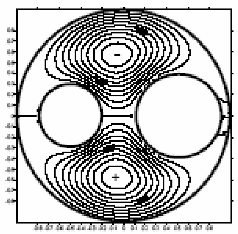
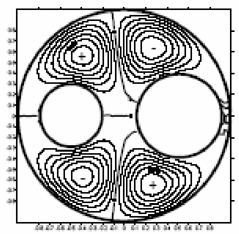
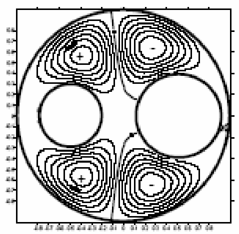
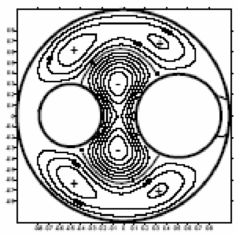
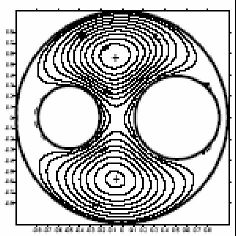
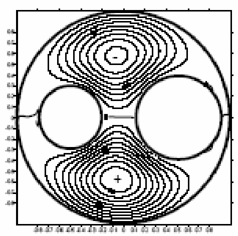
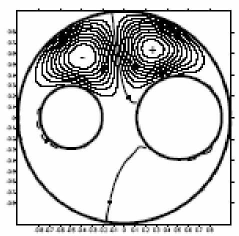
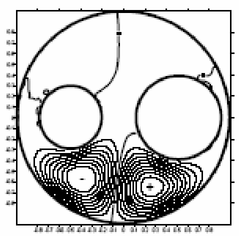
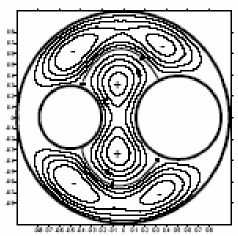
Extraction of the spurious eigenvalues by using SVD updating document



The former five eigenvalues for a multiply-connected problem with two unequal holes using different approaches

Method \ k_i	k_1	k_2	k_3	k_4	k_5
Burton & Miller method	4.82	4.82	6.72	6.72	7.82
Direct BEM + SVD Updating	4.81	4.81	6.73	6.73	7.81
Null-field BEM + SVD Updating	4.81	4.81	6.73	6.73	7.82
Fictitious BEM + SVD Updating	4.80	4.80	6.72	6.72	7.79
Direct BEM + CHIEF method	4.81	4.81	6.73	6.73	7.82
Null-field BEM + CHIEF method	4.83	4.83	6.74	6.74	7.84
Fictitious BEM + CHIEF method	4.77	4.77	6.68	6.68	7.88
FEM	4.790	4.801	6.619	6.634	7.797
Present method	4.85	4.85	6.77	6.77	7.91

The former five modes for a circle domain with two unequal holes using present method, BEM and FEM

Method \ Mode	1	2	3	4	5
Present method					
	$k = 4.85$	$k = 4.85$	$k = 6.77$	$k = 6.77$	$k = 7.91$
BEM					
	$k = 4.82$	$k = 4.82$	$k = 6.72$	$k = 6.72$	$k = 7.82$
FEM					
	$k = 4.79$	$k = 4.80$	$k = 6.62$	$k = 6.63$	$k = 1.80$

Summary

- Spurious eigenvalues depend on formulation (singular or hyper-singular).
- Spurious eigenvalues are independent of B.C. (Dirichlet or Neumann).
- Spurious eigenvalues happens to be the true eigenvalues of the interior problem (Dirichlet→singular, Neumann→hypersingular).
- To overcome the spurious eigenvalues → Burton&Miller, SVD updating term, SVD updating document…….

Conclusions

- Exterior acoustic problems (radiation and scattering) were solved by using adaptive BEM.
- Good accuracy and efficiency of the present method were obtained in comparison with those with FEM.
- Spurious eigenvalues embedded in the BIEM/BEM were examined and filtered out in this study.
- Both the fictitious frequency and spurious eigenvalue depend on the formulation instead of B.C. .

歡迎參觀海洋大學力學聲響振動實驗室

烘培雞及捎來伊妹兒

The end

Thanks for your kind attention

URL: http://ind.ntou.edu.tw/~msvlab/

Email: jtchen@mail.ntou.edu.tw

- 最新消息
- 指導教授
- 成員介紹
- 邊界元素
- 程式集錦
- 研究成果
- 大學專題
- 課程相關
- 好站介紹
- 線上期刊
- 相關文獻
- 生活剪影
- 工作月曆
- 訪客留言

您是第 014489 位訪客. (Since 2003.09.17)

如有任何意見請 E-mail 告訴我們，本網頁負責人：陳正宗，沈文成，高政宏，陳柏源，吳安傑

網頁內容部分檔案格式為*.PDF，請安裝Acrobat Reader來瀏覽

04/11/14	工程數學(三)11/19下午一點十分於河二館504舉行期中考可攜帶A4筆記一張，請攜帶學生證以備查驗!
04/11/10	恭賀 劉進賢 教授榮獲93年度國科會傑出研究獎
04/11/06	工程數學(一) A、B班聯合考試將於11/13星期六上午九點舉行，請攜帶學生證以備查驗
04/11/02	11月2日 陳正宗 特聘教授受邀於本系海工組專題演講"淺談英文科技論文寫作"
04/10/27	本實驗室最新SCI論文發表數成長曲線圖(1994~2004)
04/10/07	中華技術學院土木系十月十八日邀請陳正宗特聘教授演講"淺談英文科技論文寫作"
04/09/24	國內七所研究型大學論文發表概況分析
04/09/23	陳正宗 教授 11月19日承中山大學海下技術研究所之邀，進行專題演講"BEM in Computational Acoustics"
04/09/19	恭賀 翁煥昌 學長喜獲千金
04/09/17	恭賀 陳正宗 教授獲聘海洋大學特聘教授(第一屆)
04/08/20	海洋大學教研討會9/16舉行-陳正宗教授主講教學研究經驗分享