

考試科目	開課系級	考試日期	印製份數	答案紙	命題教師	備註
工程數學(一)	二 A,B	11月18日	111	<input checked="" type="checkbox"/> 需 <input type="checkbox"/> 不需	陳桂鴻 呂學育	第二次期中考 共1頁

- (a) Determine the complementary solution, the particular solution and general solution of a nonhomogeneous linear DE  $y^{(4)} + y''' = 1 - x^2 e^{-x}$ . (10%)

(b) Find the general solution of  $x^4 y'' + x^3 y' - 4x^2 y = 1$ , given that  $y_1(x) = x^2$  is a solution of the associated homogeneous equation. (5%)
- Cauchy-Euler equation  $x^2 y'' + 10xy' + 8y = 0$ . Find the complementary solution by using

(a) Change of variable method ( $t = \ln x$ ). (8%)

(b) Let  $y = x^m$  and looks for  $m$ . (7%)

(c) If we has found one solution,  $y_1(x) = x^{-1}$ , find  $y_2$  by using reduction order method. (10%)
- Solve the singular solution and general solution of the Clairauts equation  $y = x \frac{dy}{dx} + f\left(\frac{dy}{dx}\right)$ , where

$$f\left(\frac{dy}{dx}\right) = -e^{2y}. \text{ (10\%)}$$
- Consider (a), (b), (c), (d), determine whether the given set of function is linearly dependent or linearly independent on the interval  $(0, \infty)$ .

(a)  $f_1(x) = \cos(2x)$ ,  $f_2(x) = \sin(2x)$  (3%)

(b)  $f_1(x) = e^{2x}$ ,  $f_2(x) = e^{-2x}$  (3%)

(c)  $f_1(x) = x^2$ ,  $f_2(x) = x + 1$  (3%)

(d)  $f_1(x) = 1 + x$ ,  $f_2(x) = -x$ ,  $f_3(x) = -x^2$  (3%)

(e) is the differential equation  $(y^2 - 1)dx = xdy$  linear or nonlinear in  $x$ ? why? (3%)
- Given second-order differential equations on the interval  $(-\infty, 0)$

(1)  $x^2 y'' - 2xy' + 2y = 5x^3 \cos 3x$

(a) solve the associated homogeneous equation ( $y_c$ ) (4%)

(b) find the particular solution of the nonhomogeneous equation ( $y_p$ ) (6%)

(2)  $x^2 y'' - 3xy' + 4y = 0$ ,  $y(-1) = 2$ ,  $y'(-1) = 4$ . Find the solution of the differential equation (8%).
- Solve the given initial-value problem

$$\frac{d^2 x}{dt^2} + \beta \frac{dx}{dt} + 4x = \cos(2t); \quad x(0) = 0, x'(0) = 0$$

(a) if  $\beta = 0$  find the solution of the differential equation (5%)

(b) if  $\beta = 5$  find the solution of the differential equation (7%)

(c) if  $t \rightarrow \infty$  compare your solutions a) and b) ( $x \rightarrow \infty$  ?,...) (5%)