HOMEWORK #10 (Chapter 3 Higher –Order Differential Equations)

1. A mass of 1 slug, when attached to a spring, stretches it 2 free and then comes to rest in the equilibrium position. Determine the equation of motion if the external force is $f(t) = e^{-t}\sin 4t$. Analyze the displacements for $t \to \infty$. (Exercises 3.8 Problem 32)

Ans: $x'' + 8x' + 16x = e^{-t}\sin(4t)$, $x(0) = 0$, $x'(0) = 0$

$$x_c = c_1 e^{-4t} + c_2 t e^{-4t}, \quad x_p = -\frac{24}{625} e^{-t}\cos(4t) - \frac{7}{625} e^{-t}\sin(4t)$$

2. (a) Show that the solution of the initial-value problem $\dfrac{d^2x}{dt^2} + \omega^2 x = F_0 \cos \gamma t$, $x(0) = 0$, $x'(0) = 0$ is $x(t) = \dfrac{F_0}{\omega^2 - \gamma^2}(\cos \gamma t - \cos \omega t)$.

Ans: $x'' + \omega^2 x = F_0 \cos \gamma t$, $x_c = c_1 \cos(\omega t) + c_2 \sin(\omega t)$, $x_p = \dfrac{F_0 \cos rt}{(\omega^2 - \gamma^2)}$

(b) Evaluate $\lim\limits_{\gamma \to \omega} \dfrac{F_0}{\omega^2 - \gamma^2}(\cos \gamma t - \cos \omega t)$. (Exercises 3.8 Problem 39)

Ans: $\lim\limits_{\gamma \to \omega} \dfrac{F_0}{\omega^2 - \gamma^2}(\cos \gamma t - \cos \omega t) = \lim\limits_{\gamma \to \omega} \dfrac{-F_0 t \sin \gamma t}{-2\gamma} = \dfrac{F_0}{2\omega} t \sin \omega t$

3. Find the steady-state charge and the steady-state current in an *LRC* series circuit when $L = 1h$, $R = 2\Omega$, C=0.25f, and $E(t) = 50\cos tV$. (Exercises 3.8 Problem 49)

Ans: $q'' + 2q' + 4q = 0$, $q_c = e^{-t}(\cos\sqrt{3}t + \sin\sqrt{3}t)$

Let $q_p = A\cos t + B\sin t$, $A = \dfrac{150}{13}$, $B = \dfrac{100}{13}$

The steady-state charge is $q_p = \dfrac{150}{13}\cos t + \dfrac{100}{13}\sin t$

The steady-state current is $i_p = -\dfrac{150}{13}\sin t + \dfrac{100}{13}\cos t$