HOMEWORK #11 (Chapter 3 Higher –Order Differential Equations)

In problem, solve equation (4) subject to the appropriate boundary conditions. The beam is of length L and ω_0 is a constant. (where equation (4) is $EI\dfrac{d^4y}{dx^4}=\omega(x)$)

1. The beam is embedded at its left end and simply supported at its right end and $\omega(x)=\omega_0\sin(\frac{\pi x}{L})$, $0<x<L$. (Exercises 3.9 Problem 4)

Ans: $EIy^{(4)}=\omega_0\sin(\frac{\pi x}{L})$

$$y_c=c_1+c_2x+c_3x^2+c_4x^3$$

Let $y_p=a\sin(\frac{\pi x}{L})+b\cos(\frac{\pi x}{L})$, $y_p^{(4)}=(\frac{\pi}{L})^4a\sin(\frac{\pi}{L}x)+(\frac{\pi}{L})^4b\cos(\frac{\pi}{L}x)$

$$(\frac{\pi}{L})^4a\sin(\frac{\pi}{L}x)+(\frac{\pi}{L})^4b\cos(\frac{\pi}{L}x)=\frac{\omega_0}{EI}\sin(\frac{\pi x}{L}),\quad \therefore a=\frac{\omega_0}{EI}(\frac{L}{\pi})^4,\quad b=0$$

$$y_p=\frac{\omega_0}{EI}(\frac{L}{\pi})^4\sin(\frac{\pi x}{L})$$

$$y=c_1+c_2x+c_3x^2+c_4x^3+\frac{\omega_0}{EI}(\frac{L}{\pi})^4\sin(\frac{\pi x}{L})$$

Boundary conditions are $y(0)=0$, $y'(0)=0$, $y(L)=0$, $y''(L)=0$

$$\therefore c_1=0,\ c_2=\frac{-\omega_0}{EI}(\frac{L}{\pi})^3,\ c_3=\frac{3\omega_0L^2}{2EI\pi^3},\ c_4=\frac{-\omega_0L}{2EI\pi^3}$$

$$\therefore y=\frac{\omega_0L}{2EI\pi^3}[-2L^2x+3Lx^2-x^3+\frac{2L^3}{\pi}\sin(\frac{\pi x}{L})]$$