

HOMEWORK #2 Answers (Chapter 2 First –Order Differential Equations)

In problem 1-4, solve the given differential equation by separation of variables.

$$(1) \frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

$$\text{Ans: } \frac{dy}{dx} = \frac{1 - (\frac{y}{x})^2}{2(\frac{y}{x})}, \quad \text{令 } u = \frac{y}{x}, \quad y' = u + u'x$$

$$\therefore u + u'x = \frac{1 - u^2}{2u} \rightarrow x \frac{du}{dx} = \frac{1 - 3u^2}{2u} \rightarrow \frac{2u}{1 - 3u^2} du = \frac{1}{x} dx \rightarrow \frac{-1}{3} \ln \left| 1 - 3\left(\frac{y}{x}\right)^2 \right| = \ln|x| + c_1$$

$$(2) e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y} \quad (\text{problem 8})$$

$$\text{Ans: } ye^y dy = (e^{-x} + e^{-3x}) dx \rightarrow ye^y - e^y = -e^{-x} - \frac{1}{3}e^{-3x} + c_1$$

$$(3) (e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0 \quad (\text{problem 13})$$

$$\text{Ans: } \frac{e^x}{(e^x + 1)^3} dx = -\frac{e^y}{(e^y + 1)^2} dy \rightarrow \frac{-1}{2}(e^x + 1)^{-2} = \frac{1}{(e^y + 1)} + c_1$$

$$(4) \frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8} \quad (\text{problem 19})$$

$$\text{Ans: } \frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8} = \frac{(y+3)(x-1)}{(y-2)(x+4)} \rightarrow \frac{(y-2)}{(y+3)} dy = \frac{(x-1)}{(x+4)} dx$$

$$\left(1 - \frac{5}{y+3}\right) dy = \left(1 - \frac{5}{x+4}\right) dx \rightarrow y - 5 \ln|y+3| = x - 5 \ln|x+4| + c_1$$