HOMEWORK #3 (Chapter 2 First –Order Differential Equations)

In problem 1, solve the given initial-value problem.

(1) $y^{1/2} \dfrac{dy}{dx} + y^{3/2} = 1$, $y(0) = 4$

In problem 2, solve the given Bernoulli equation by using an appropriate substitution

(2) $x \dfrac{dy}{dx} + y = \dfrac{1}{y^2}$

(3) The differential equation $\dfrac{dy}{dx} = P(x) + Q(x)y + R(x)y^2$ is known as Ricatti's equation.

(a) A Ricatti equation can be solved by a succession of two substitutions provided we know a particular solution y_1 of the equation. Show that the substitution $y = y_1 + u$ reduces Ricatti's equation to a Bernoulli equation (4) with $n = 2$. The Bernoulli equation can that be reduced to a linear equation by the substitution $w = u^{-1}$.

(b) Find a one-parameter family of solutions for the differential equation

$$\dfrac{dy}{dx} = -\dfrac{4}{x^2} - \dfrac{1}{x}y + y^2 .$$