HOMEWORK #4 (Chapter 2 First –Order Differential Equations)

1. Suppose $M(x,y)dx + N(x,y)dy = 0$ is a nonexact differential equation. Under certain conditions integrating factors (see Problem 29 and 30) can be found in a straightforward manner. If $(M_Y - N_x)/N = p(x)$, that is a function of x alone, then an integrating factor is $\mu(x) = e^{\int p(x)dx}$. If $(N_x - M_y)/M = q(y)$, that is a function of y alone, then an integrating factor is $\mu(y) = e^{\int q(y)dy}$. In Problem (a),(b) (33,34) ,solve the given differential equation by finding an appropriate integrating factor.

(a) $6xydx + (4y + 9x^2)dy = 0$

(b) $\cos xdx + (1 + \frac{2}{y})\sin xdy = 0$

2. Solve $(y')^2 - xy' + y = 0$ and plot the graph of general solution and singular solution.

3. A Bernoulli equation of $y' + xy = xy^2$,

(a) Homeogeneous (Yes or No), why?

(b) O.D.E or P.D.E, why?

(c) Linear or nonlinear, why?

(d) First-order or Second-order, why?

(e) Separable (Yes or No), why?

(f) Exact (Yes or No), why?

(g) Solve by Separable variable method.

(h) Solve by linear O.D.E method (Convert the O.D.E to a linear equation by using the change of variable method).

(i) Solve by the Exact O.D.E method.

4. The Ricatti equation $y' = \frac{1}{2x}y^2 - \frac{1}{x}y - \frac{4}{x}$ by using the solution $y_2 = y_1 + \frac{1}{z}$ with $y_1 = 4$, we obtain $y_2 = 4 + \frac{6x^3}{C - x^3}$, $C \in R$.

(a) By setting C=0, we have $y_2 = -2$, solve $y_3 = -2 + \frac{1}{z}$, please find y_3.

(b) $y_3 = y_2$, Yes or No?

(c) The Ricatti equation is linear or nonlinear?