HOMEWORK #5 (Chapter 3 Higher –Order Differential Equations)

1. $(x-2)y''-(4x-7)y'+(4x-6)y=0$, $y_1=e^{2x}$, Find the general solution.

Ans: $y''-\dfrac{(4x-7)}{(x-2)}y'+\dfrac{(4x-6)}{(x-2)}y=0$, $p(x)=-\dfrac{(4x-7)}{(x-2)}=-\left[4+\dfrac{1}{(x-2)}\right]$

$$e^{-\int p(x)dx}=e^{4x+\ln|x-2|}=(x-2)e^{4x},\quad y_2=y_1\int\frac{e^{-\int p(x)dx}}{y_1^2}dx=e^{2x}(\frac{x^2}{2}-2x)$$

$$y=c_1y_1+c_2y_2=c_1e^{2x}+c_2e^{2x}(\frac{x^2}{2}-2x)$$

2. In Problems (a),(b), the indicted function $y_1(x)$ is a solution of the given equation. Use reduction of order or formula (5), as instructed, to find a second solution $y_2(x)$.

(a). $9y''-12y'+4y=0$, $y_1=e^{\frac{2x}{3}}$ (Problem 7)

Ans: $y''-\dfrac{4}{3}y'+\dfrac{4}{9}y=0$, $p(x)=\dfrac{-4}{3}$, $e^{-\int p(x)dx}=e^{\frac{4}{3}x}$, $y_2=y_1\int\dfrac{e^{-\int p(x)dx}}{y_1^2}dx=xe^{\frac{2}{3}x}$

(b). $x^2y''-7xy'+16y=0$, $y_1=x^4$ (Problem 9)

Ans: $y''-\dfrac{7}{x}y'+\dfrac{16}{x^2}y=0$, $p(x)=\dfrac{-7}{x}$, $e^{-\int p(x)dx}=x^7$, $y_2=y_1\int\dfrac{e^{-\int p(x)dx}}{y_1^2}dx=x^4\ln|x|$