

HOMEWORK #6 (Chapter 3 Higher -Order Differential Equations)

1. Find the general solution of the given higher-order differential equation

$$y''' + 3y'' + 3y' + y = 0 \quad (\text{Problem 21})$$

Ans: $\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0, (\lambda + 1)(\lambda + 1)^2 = 0, \lambda = -1, -1, -1$

$$\therefore y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$$

2. solve the given boundary-value problem.

$$y'' - 2y' + 2y = 0, \quad y(0) = 1, \quad y(\pi) = 1 \quad (\text{Problem 42})$$

Ans: $\lambda^2 - 2\lambda + 2 = 0, \lambda = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i, \therefore y = e^x [c_1 \cos x + c_2 \sin x]$

$$y(0) = 1 \Rightarrow c_1 = 1, \quad y(\pi) = 1 \Rightarrow 1 \neq e^\pi$$

\therefore Since $1 \neq e^\pi$ the boundary-value problem has no solution.