

HOMEWORK #7 (Chapter 3 Higher –Order Differential Equations)

Solve the given differential equation.

1. $x^2 y'' + xy' + 4y = 0$ (Exercises 3.6 Problem 5)

Ans: Let $y = x^m \Rightarrow m(m-1) + m + 4 = 0 \Rightarrow m = \pm 2i$

$$\therefore y = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)$$

2. $x^3 y''' + xy' - y = 0$ (Exercises 3.6 Problem 16)

Ans: Let $y = x^m \Rightarrow m(m-1)(m-2) + m - 1 = 0 \Rightarrow m = 1, 1, 1$

$$\therefore y = c_1 x + c_2 x \ln x + c_3 x (\ln x)^2$$

Solve the given initial-value problem.

3. $x^2 y'' + xy' + y = 0$, $y(1) = 1$, $y'(1) = 2$ (Exercises 3.6 Problem 25)

Ans: Let $y = x^m \Rightarrow m(m-1) + m + 1 = 0 \Rightarrow m = \pm i$

$$\therefore y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$$

$$y(1) = 1 \Rightarrow c_1 = 1$$

$$y' = -c_1 \sin(\ln x) \frac{1}{x} + c_2 \cos(\ln x) \frac{1}{x}$$

$$y'(1) = 2 \Rightarrow c_2 = 2$$

$$\therefore y = \cos(\ln x) + 2 \sin(\ln x)$$

Solve the given differential equation by undetermined coefficients.

4. $y'' + 3y = -48x^2 e^{3x}$ (Exercises 3.4 Problem 7)

Ans: Let $y_c = e^{\lambda x} \Rightarrow \lambda^2 + 3 = 0 \Rightarrow \lambda = \pm \sqrt{3}i$, $\therefore y_c = c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)$

$$\text{Let } y_p = (ax^2 + bx + c)e^{3x}$$

$$y_p'' = [9ax^2 + (12a + 9b)x + (2a + 6b + 9c)]e^{3x}$$

$$\therefore [12ax^2 + (12a + 12b)x + (2a + 6b + 12c)]e^{3x} = -48x^2 e^{3x}$$

$$\Rightarrow a = -4, b = 4, c = -\frac{4}{3}$$

$$y_p = \left(-4x^2 + 4x - \frac{4}{3}\right)e^{3x}$$

$$y = y_c + y_p = c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x) + \left(-4x^2 + 4x - \frac{4}{3}\right)e^{3x}$$