HOMEWORK #9 (Chapter 3 Higher –Order Differential Equations)

In problem 1-4, solve the given differential equation by using the substitution $u = y'$.

1. $y'' + (y')^2 + 1 = 0$ (Exercises 3.7 Problem 3)

Ans: Let $u = y'$, $u' = y''$

$$u' = -(1+u^2) \rightarrow \frac{1}{u^2+1}du = -dx \rightarrow \tan^{-1}(u) = -x + c_1 \rightarrow y' = \tan(c_1 - x) \rightarrow$$

$$dy = \tan(c_1 - x)dx \rightarrow \therefore y = \ln|\cos(c_1 - x)| + c_2$$

2. $y'' = 1 + (y')^2$ (Exercises 3.7 Problem 4)

Ans: Let $u = y'$, $u' = y''$

$$u' = 1 + u^2 \rightarrow \frac{1}{u^2+1}du = dx \rightarrow \tan^{-1}(u) = x + c_1 \rightarrow y' = \tan(c_1 + x) \rightarrow$$

$$dy = \tan(c_1 + x)dx \rightarrow \therefore y = -\ln|\cos(c_1 + x)| + c_2$$

3. $x^2 y'' + (y')^2 = 0$ (Exercises 3.7 Problem 5)

Ans: Let $u = y'$, $u' = y''$

$$x^2 u' + u^2 = 0 \rightarrow \frac{-1}{u^2}du = \frac{1}{x^2}dx \rightarrow \frac{1}{u} = \frac{-1}{x} + c_1 \rightarrow y' = \frac{x}{c_1 x - 1} \rightarrow y' = \frac{1}{c_1}(\frac{1}{c_1 x + 1} - 1)$$

$$\therefore y = \frac{1}{c_1^2}\ln|c_1 x + 1| - \frac{1}{c_1}x + c_2$$

4. $(y+1)y'' = (y')^2$ (Exercises 3.7 Problem 6)

Ans: Let $u = y'$, $y'' = \frac{d}{dx}(y') = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = \frac{du}{dy}u$

$$(y+1)u\frac{du}{dy} = u^2 \rightarrow \frac{1}{u}du = \frac{1}{(y+1)}dy \rightarrow \ln|u| = \ln|y+1| + \ln|c_1| \rightarrow y' = c_1(y+1) \rightarrow$$

$$\frac{1}{(y+1)}dy = c_1 dx \rightarrow \ln|y+1| = c_1 x + c_2 \rightarrow y + 1 = e^{c_1 x}e^{c_2}$$

$$\therefore y = c_3 e^{c_1 x} - 1$$