Final Exam

Jan. 2005

1) Solve $4x^2y'' + 2xy' - xy = 0$ by using the method of **Frobenius** (25 scores)

a) show that zero is a regular singular point of the differential equation

b) solve the indicial equation

c) determine the recurrence relation

d) find the Frobenius solution (the **first five** terms) based on r_2, the smaller root of the indicial equation

2) Solve $x^2y'' + 2xy' - 6y = 0$ by using the method of **Frobenius** (25 scores)

a) point out all the singular point(s) of the differential equation

b) solve the indicial equation

c) determine the recurrence relation

d) find the general solution using the results of a) and b)

3) a) Let $L[f(t)](s) = F(s)$, show $L[t\ f(t)](s) = -F'(s)$ (hint: $L[f](s) = \int_0^\infty e^{-st} f(t)dt$)

(10 scores)

b) Known $L[f'](s) = sF(s) - f(0)$, show $L[f''](s) = s^2F(s) - sf(0) - f'(0)$ (10 scores)

c) what is the Dirac Delta function $\delta(t)$ (5 scores)

d) Solve $y'' + 2ty' - 2y = \delta(t);\quad y(0) = y'(0) = 0$ by applying the Laplace transform

(hint: $\lim_{s\to\infty} F(s) = 0$) (20 scores)

4) Consider $y'' + 4y' + 4y = f(t),\quad y(0) = 0,\quad y'(0) = 0$

$$f(t) = \begin{cases} 1 & 0 \le t < 2 \\ 0 & t \ge 2 \end{cases} \quad for$$

a) plot the graph of $f(t)$ (5 scores)

b) what is the Heaviside function $H(t)$ (5 scores)

c) describe $f(t)$ in terms of the Heaviside function (5 scores)

d) solve the initial value problem by using the Laplace transform (20 scores)