工程數學二 B 班期末考參考解答 Jan.14,2004 (J.T. Chen)

1. Please fill in the following table (20%)

	$p(x)$	$q(x)$	$x=0$ Regular? Irregularly singular? Regularly singular?	Indicial equation?	r_1 r_2
$(1-x^2)y''-2xy'+2y=0$	$\frac{-2x}{1-x^2}$	$\frac{2}{1-x^2}$	Regular	X	X
$y''-\frac{2}{(1-x)^2}y=0$	0	$\frac{-2}{(1-x)^2}$	Regular	X	X
$4xy''+2y'+y=0$	$\frac{1}{2x}$	$\frac{1}{4x}$	Regularly singular	$r^2-\frac{1}{2}r=0$	$r=0$, $r=\frac{1}{2}$
$x^2y''+xy'+x^2y=0$	$\frac{1}{x}$	1	Regularly singular	$r^2=0$	0 , 0

where $y''+p(x)y'+q(x)y=0$

2. Solve the series solution of $(1-x^2)y''-2xy'+6y=0$. (20%)

Ans: $P_2(x)=1-3x^2$, $y_2=x-\frac{2}{3}x^3-\frac{1}{5}x^5-\cdots$

3. Solve the indicial equation and series solution of $x^2y''-4xy'-6y=0$ using $y(x)=\sum_{n=0}^{\infty}c_nx^{n+r}$. (20%)

Ans: $r^2-5r-6=0$, $r=6\ or\ -1$, $x^6\ or\ x^{-1}$

4. Find the indicial equation of $x(x-1)y''+3xy'+y=0$. (10%)

Find the series solution for the case of smaller r_1. (10%)

Ans: $r^2-r=0$, $a_n=(\frac{n}{n-1})a_{n-1}$, $y=c_1(x+2x^2+\frac{8}{3}x^3+\frac{10}{3}x^4+\cdots)$

5. Find the indicial equation of $(1-x^2)y''-xy'+y=0$. (10%)

Find the series solution using $\sum_{n=0}^{\infty}c_nx^{n+r}$ for the case of larger r_2. (10%)

Ans: $r^2-r=0$, $y=c_0+c_1(x+\frac{1}{4}x^3+\frac{1}{8}x^5+\frac{5}{64}x^7+\cdots)$

6. Find the indicial equation of $(1-x^2)y''-xy'+4y=0$. (10%)

Find the series solution using $\sum_{n=0}^{\infty}c_nx^{n+r}$ for the case of smaller r_1. (10%)

Ans: $r^2-r=0$, $a_{n+2}=\frac{n^2-4}{(n+1)(n+2)}a_n$, $y=c_0(1-2x^2)+c_1(x-\frac{1}{2}x^3-\frac{1}{8}x^5-\frac{1}{16}x^7+\cdots)$

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