Engineering Mathematics I HW#10 日河工 2B

1)* *Sections 3.3 Problems* 3. (p. 139)

$$f(t)=\begin{cases} 1 & \text{for } 0\le t<7 \\ \cos(t) & t\ge 7 \end{cases}$$

==> $f(t)=[H(t)-H(t-7)]+\cos(t)H(t-7)=[H(t)-H(t-7)]+\cos(t-7+7)H(t-7)$

$=[H(t)-H(t-7)]+[\cos(7)\cos(t-7)-\sin(7)\sin(t-7)]H(t-7)$

➔ $\mathcal{L}[f(t)]=\frac{1}{s}[1-e^{-7s}]+\frac{s}{s^2+1}\cos(7)e^{-7s}-\frac{1}{s^2+1}\sin(7)e^{-7s}$

2) *Sections 3.3 Problems* 9. (p. 139)

$$f(t)=\begin{cases} \cos(t) & \text{for } 0\le t<2\pi \\ 2-\sin(t) & t\ge 2\pi \end{cases}$$

➔ $f(t)=\cos(t)[H(t)-H(t-2\pi)]+[2-\sin(t)]H(t-2\pi)$

$=\cos(t)H(t)-\cos(t-2\pi+2\pi)H(t-2\pi)+2H(t-2\pi)-\sin(t-2\pi+2\pi)H(t-2\pi)$

➔ $=\cos(t)H(t)-[\cos(t-2\pi)\cos(2\pi)-\sin(t-2\pi)\sin(2\pi)]H(t-2\pi)+2H(t-2\pi)-$

$[\sin(t-2\pi)\cos(2\pi)+\cos(t-2\pi)\sin(2\pi)]H(t-2\pi)$

➔ $=\cos(t)H(t)-\cos(t-2\pi)H(t-2\pi)+2H(t-2\pi)-\sin(t-2\pi)H(t-2\pi)$

➔ $\mathcal{L}[f(t)]=\frac{s}{s^2+1}+[\frac{2}{s}-\frac{s}{s^2+!}-\frac{1}{s^2+1}]e^{-2\pi s}$

3) *Section 3.3 Problems* 33. (p. 140)

$y^{(3)}-8y=g(t);\quad y(0)=y'(0)=y''(0)=0$

$$g(t)=\begin{cases} 0 & \text{for } 0\le t<6 \\ 2 & t\ge 6 \end{cases}$$

➔ $y^{(3)}-8y=2H(t-6)$

➔ $s^3Y(s)-s^2y(0)-sy'(0)-y''(0)-8Y(s)=\frac{2}{s}e^{-6s}$

➔ $Y(s)[s^3-8]=\frac{2}{s}e^{-6s},\quad Y(s)=\frac{2}{s(s^3-8)}e^{-6s}$

➔ $Y(s)=\left[\frac{-1}{4s}+\frac{1}{12}\left(\frac{1}{s-2}\right)+\frac{1}{6}\left(\frac{s+1}{s^2+2s+4}\right)\right]e^{-6s}$

➔ $y(t)=\left\{-\frac{1}{4}+\frac{1}{12}e^{2(t-6)}+\frac{1}{6}e^{-(t-6)}\cos[\sqrt{3}(t-4)]\right\}H(t-6)$

4) *Section 3.3 Problems* 37. (p. 140)

$$y'' + 2y' - 7y = f(t), \quad y(0) = -2, \quad y'(0) = 0$$

$$f(t) = \begin{cases} 0 & \text{for } 0 \le t < 5 \\ 2 & \quad t \ge 5 \end{cases}$$

➔ $y'' + 2y' - 7y = 2H(t-5)$

➔ $Y(s) = \dfrac{-2s}{s^2 + 2s - 7} + \dfrac{2}{s(s^2 + 2s - 7)} e^{-5s}$

➔ $y(t) = -\dfrac{1}{4}\left[(4-\sqrt{2})e^{-(1+2\sqrt{2})t} + (4+\sqrt{2})e^{-(1-2\sqrt{2})t}\right] - \dfrac{1}{28}\left[8 - (4-\sqrt{2})e^{-(1+2\sqrt{2})(t-5)} - (4+\sqrt{2})e^{-(1-2\sqrt{2})(t-5)}\right]H(t-5)$

5) *Section 3.3 Problems* 39. (p.140)

$$y'' + 4y' + 4y = f(t), \quad y(0) = 1, \quad y'(0) = 2$$

$$f(t) = \begin{cases} 1 & \text{for } 0 \le t < 2 \\ 0 & \quad t \ge 2 \end{cases}$$

➔ $y'' + 4y' + 4y = H(t) - H(t-2)$

➔ $s^2Y(s) - sy(0) - y'(0) + 4Y(s) - 4y(0) + 4Y(s) = \dfrac{1}{s} - \dfrac{e^{-2s}}{s}$

➔ $Y(s) = \dfrac{s+6}{(s+2)^2} + \dfrac{1}{s(s+2)^2}(1 - e^{-2s})$

➔ $Y(s) = \dfrac{1}{4s} + \dfrac{3}{4(s+2)} + \dfrac{7}{2(s+2)^2} - \left[\dfrac{1}{4s} - \dfrac{1}{4(s+2)} - \dfrac{1}{2(s+2)^2}\right]e^{-2s}$

➔ $y(t) = \dfrac{1}{4} + \dfrac{3}{4}e^{-2t} + \dfrac{7}{2}te^{-2t} + \left[-\dfrac{1}{4} + \dfrac{1}{4}e^{-2(t-2)} + \dfrac{1}{2}(t-2)e^{-2(t-2)}\right]H(t-2)$

6) *Section 3.3 Problems* 47. (p.141)

$$f(t) = K[H(t-a) - H(t-b)]$$

➔ $\mathcal{L}[f(t)] = \dfrac{K}{s}\left[e^{-as} - e^{-bs}\right]$

7) *Section 3.3 Problems* 49. (p.141)

$$f(t) = \frac{h}{b-a}(t-a)[H(t-a) - H(t-b)] + \frac{h}{b-c}(t-c)[H(t-b) - H(t-c)]$$

➔ $f(t) = \dfrac{h}{b-a}(t-a)H(t-a) + \dfrac{h(c-a)}{(b-c)(b-a)}(t-b)H(t-b) + \dfrac{h}{b-c}(t-c)H(t-c)$

➔ $\mathcal{L}[f(t)] = \dfrac{h}{s^2}\left[\dfrac{e^{-as}}{b-a} + \dfrac{c-a}{(b-c)(b-a)}e^{-bs} - \dfrac{e^{-cs}}{b-c}\right]$