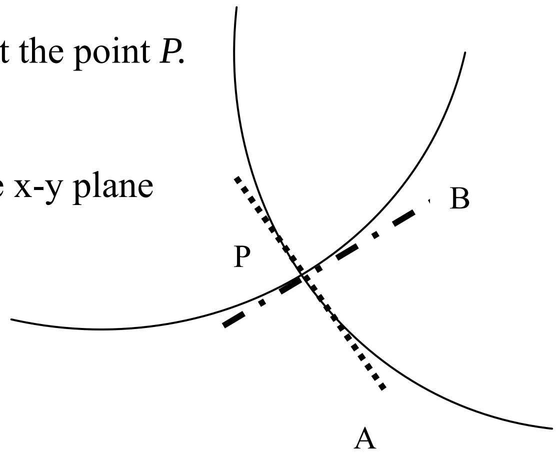
Engineering Mathematics I　HW#2　　日河工 2B

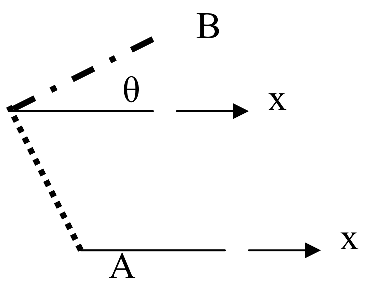
1) Given a family F of curves $x^2 - Ky = 1$, a) show that the slopes of two lines, orthogonal to each other, are negative reciprocals, b)describe the definition of orthogonal trajectories of a given family of curves, c)find the family of orthogonal trajectories of the given family F of curves.

a) As shown on the right, two curves intersect at the point *P*. Line *A* and Line *B* are their tangents at *P*.



The slope of Line B is $y_B' = \frac{dy}{dx} = \tan\theta$ in the x-y plane

With



The slope of Line A is $y_A' = \frac{dy}{dx} = \tan(\theta + \pi/2) = \sin(\theta+\pi/2)/\cos(\theta+\pi/2)$

$$= \frac{[\sin(\theta)\cos(\pi/2) + \cos(\theta)\sin(\pi/2)]}{[\cos(\theta)\cos(\pi/2) - \sin(\theta)\sin(\pi/2)]}$$

$$= \frac{[\cos(\theta)]}{[-\sin(\theta)]} = -1/\tan(\theta)$$

$$\therefore y'_A y'_B = -1$$

b) Two families of curves, or trajectories, are orthogonal if each curve of the first family is orthogonal to each curve of the second family wherever an intersection occurs.

c) $x^2 - Ky = 1$ ➔ $d(x^2 - Ky)/dx = d(1)/dx$ ➔ $2x - Ky' = 0$

➔ $y' = \frac{2x}{K} = \frac{2xy}{x^2 - 1}$,

Orthogonal trajectories $y' = -\frac{x^2-1}{2xy}$ ➔ $y^2 = \ln|x| - \frac{x^2}{2} + C$

2) A 16-meter-long chain ***weighing*** ρ kg per meter hangs over a small pulley (as shown in Fig. 1.16 of the textbook), which is 20 meters above the floor. Initially, the chain is held at rest, with 7 meters on one side and 9 meters on the other side. How long after the chain is released, and with what velocity, a) as it becomes 4 meters on one side and 12 meters on the other side, b) will it leave the pulley ?

It will be detailed in the class.

3) Solve the Problem 3. in *Section 1.5 Problems* of the textbook.

Ans: Consider $y - xy' = 0$

a) $M(x,y) = y,\ \ N(x,y) = -x$

$$\frac{\partial M}{\partial y} = \frac{\partial y}{\partial y} = 1,\ \ \frac{\partial N}{\partial x} = \frac{\partial(-x)}{\partial x} = -1 \ \Rightarrow\ \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

By theorem 1.1, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ ==> $y - xy' = 0$ is not exact.

b) $\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x} \Rightarrow \frac{\partial \mu}{\partial y} M + \mu \frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x} N + \mu \frac{\partial N}{\partial x}$

➔ For $\mu(x)$ is a function of x alone, $\frac{\partial \mu}{\partial y} = 0$

➔ $\mu(x) = -x\frac{d\mu(x)}{dx} - \mu(x),\ \ 2\mu(x) + x\frac{d\mu(x)}{dx} = 0$

➔ $-\frac{d\mu}{\mu} = 2\frac{dx}{x},\ \ -\ln|\mu| = 2\ln|x| + C,\ \ \mu^{-1} = x^2$ (with C=0)

➔ $\mu = \frac{1}{x^2}$

c) $\frac{\partial(vM)}{\partial y} = \frac{\partial(vN)}{\partial x} \Rightarrow \frac{\partial v}{\partial y} M + v \frac{\partial M}{\partial y} = \frac{\partial v}{\partial x} N + v \frac{\partial N}{\partial x}$

➔ For $v(y)$ is a function of y alone, $\frac{\partial v}{\partial x} = 0$

➔ $\frac{dv(y)}{dy} y + v(y) = -v(y),\ \ 2v(y) + y\frac{dv(y)}{dy} = 0$

➔ $-\frac{dv}{v} = 2\frac{dy}{y}$, $-\ln|v| = 2\ln|y| + C$, $v^{-1} = y^2$ (with C=0)

➔ $\mu = \frac{1}{y^2}$

d) $\frac{\partial(x^a y^b M)}{\partial y} = \frac{\partial(x^a y^b N)}{\partial x}$, $\frac{\partial(x^a y^b)}{\partial y} M + x^a y^b \frac{\partial M}{\partial y} = \frac{\partial(x^a y^b)}{\partial x} N + x^a y^b \frac{\partial N}{\partial x}$

$bx^a y^{b-1} y + x^a y^b = ax^{a-1} y^b (-x) - x^a y^b$, $bx^a y^b + x^a y^b = -ax^a y^b - x^a y^b$

➔ $(b+1)x^a y^b = -(a+1)x^a y^b$

for some constants a, b ➔ $(b+1) = -(a+1)$ ➔ $a + b = -2$

Note: by the general form, $\frac{\partial v}{\partial y} M + v\frac{\partial M}{\partial y} = \frac{\partial v}{\partial x} N + v\frac{\partial N}{\partial x}$

it seems that we can do something more. And we will do it in the class.