Engineering Mathematics I HW#6 日河工 2B

1) *Section 2.6 Problems* 1. (p. 97)

$y'' + y = \tan(x)$

➔ $\lambda^2 + 1 = 0, \quad \lambda = i, \quad -i$ ➔ $y_h = c_1 \cos(x) + c_2 \sin(x)$

$y_p = u(x)y_1 + v(x)y_2 = u\cos(x) + v\sin(x)$

$$W(x) = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) + \sin^2(x) = 1$$

$$u'(x) = -\frac{y_2 f(x)}{W(x)} = -\frac{\sin^2(x)}{\cos(x)} = \cos(x) - \sec(x), \quad \text{==>} \ u = \sin(x) - \ln|\sec(x) + \tan(x)|$$

$$v'(x) = \frac{y_1 f(x)}{W(x)} = \cos(x)\tan(x) = \sin(x), \quad \text{==>} \ v = -\cos(x)$$

$$y_p = u\cos(x) + v\sin(x) = \cos(x)\sin(x) - \cos(x)\ln|\sec(x) + \tan(x)| - \cos(x)\sin(x)$$

$$y = c_1\cos(x) + c_2\sin(x) + u\cos(x) + v\sin(x) = c_1\cos(x) + c_2\sin(x) - \cos(x)\ln|\sec(x) + \tan(x)|$$

2) *Section 2.6 Problems* 5. (p.97)

$y'' - 3y' + 2y = \cos(e^{-x})$

➔ $\lambda^2 - 3\lambda + 2 = 0, \quad \lambda = 1, \quad 2$ ➔ $y_h = c_1 e^x + c_2 e^{2x}$

$y_p = u(x)y_1 + v(x)y_2 = ue^x + ve^{2x}$

$$W(x) = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = 2e^{3x} - e^{3x} = e^{3x}$$

$$u'(x) = -\frac{y_2 f(x)}{W(x)} = -\frac{e^{2x}\cos(e^{-x})}{e^{3x}} = -e^{-x}\cos(e^{-x}), \quad \text{==>} \ u = \sin(e^{-x})$$

$$v'(x) = \frac{y_1 f(x)}{W(x)} = \frac{e^x \cos(e^{-x})}{e^{3x}} = e^{-2x}\cos(e^{-x}), \quad \text{==>} \ v = -\cos(e^{-x}) - e^{-x}\sin(e^{-x})$$

$$y_p = ue^x + ve^{2x} = e^x\sin(e^{-x}) - e^{2x}\cos(e^{-x}) - e^x\sin(e^{-x})$$

$$y = c_1 e^x + c_2 e^{2x} - e^{2x}\cos(e^{-x})$$

3) *Section 2.6 Problems* 23. (p. 97)

$y'' + 4y = 5\sinh(2x)$

➔ $\lambda^2 + 4 = 0, \quad \lambda = 2i, \quad -2i$ ➔ $y_h = c_1\cos(2x) + c_2\sin(2x)$

$y'' + 4y = 5\sinh(2x) = 5\dfrac{e^{2x} - e^{-2x}}{2}$

$$y_p = ae^{2x} + be^{-2x}$$
$$y_p' = 2ae^{2x} - 2be^{-2x}$$
$$y_p'' = 4ae^{2x} + 4be^{-2x}$$

➔ $4ae^{2x} + 4be^{-2x} + 4(ae^{2x} + be^{-2x}) = \dfrac{5}{2}e^{2x} - \dfrac{5}{2}e^{-2x}, \quad a = -b = \dfrac{5}{16}$

$$y_p = ae^{2x} + be^{-2x} = \frac{5}{8}\frac{e^{2x} - e^{-2x}}{2} = \frac{5}{8}\sinh(2x)$$

$$y = c_1\cos(2x) + c_2\sin(2x) + \frac{5}{8}\sinh(2x)$$

4) *Section 2.6 Problems* 25. (p.97)

$y'' + 4y' + 4y = 7x - 3\cos(2x) + 5xe^{-2x}$

➔ $\lambda^2 + 4\lambda + 4 = 0, \quad \lambda = -2, \quad -2$ ➔ $y_h = c_1e^{-2x} + c_2xe^{-2x}$

$$y_p = Ax + B + C\sin(2x) + D\cos(2x) + (Ex + F)x^2e^{-2x}$$
$$y_p' = A + 2C\cos(2x) - 2D\sin(2x) + Ex^2e^{-2x} - 2(Ex + F)x^2e^{-2x} + 2(Ex + F)xe^{-2x}$$
$$y_p'' = -4C\sin(2x) - 4D\cos(2x) + 2Exe^{-2x} - 2Ex^2e^{-2x} - 2Ex^2e^{-2x} - 4(Ex + F)xe^{-2x} +$$
$$4(Ex + F)x^2e^{-2x} + 2Exe^{-2x} + 2(Ex + F)e^{-2x} - 4(Ex + F)xe^{-2x}$$

==> $A = \dfrac{7}{4}, \quad B = -\dfrac{7}{4}, \quad C = -\dfrac{3}{8}, \quad D = 0, \quad E = \dfrac{5}{6}, \quad F = 0$

$$y = c_1e^{2x} + c_2xe^{-2x} - \frac{7}{4} + \frac{7}{4}x - \frac{3}{8}\sin(2x) + \frac{5}{6}x^3e^{-2x}$$

5) *Section 2.6 Problems* 33. (p. 97)

$$x^2 y'' + xy' + 4y = \sin(2\ln(x))$$

$$y'' + \frac{1}{x} y' + \frac{4}{x^2} y = \frac{\sin(2\ln(x))}{x^2}$$

$x = e^t$, $Y(t) = y(e^t)$

$Y'' + 4Y = 0$, $\lambda = \pm 2i$

$Y_1 = \cos(2t)$, $Y_2 = \sin(2t)$ ==> $y_1 = \cos(2\ln(x))$, $y_2 = \sin(2\ln(x))$

$$y_p = u(x)y_1 + vy_2, \qquad f(x) = \frac{\sin(2\ln(x))}{x^2}$$

$$W(x) = \begin{vmatrix} \cos(2\ln(x)) & \sin(2\ln(x)) \\ -\frac{2}{x}\sin(2\ln(x)) & \frac{2}{x}\cos(2\ln(x)) \end{vmatrix} = \frac{2}{x}\left(\cos^2(2\ln(x)) + \sin^2(2\ln(x))\right) = \frac{2}{x}$$

$$u' = -\frac{y_2 f(x)}{W(x)} = -\frac{\sin^2(2\ln(x))}{2x} = -\frac{1}{4x} + \frac{\cos(4\ln(x))}{4x}, \quad u = -\frac{1}{4}\ln(x) + \frac{\sin(4\ln(x))}{16}$$

$$v' = \frac{y_1 f(x)}{W(x)} = \frac{\cos(2\ln(x))\sin(2\ln(x))}{2x} = \frac{\sin(4\ln(x))}{4x}, \quad v = -\frac{\cos(4\ln(x))}{16}$$

$$y = c_1 \cos(2\ln(x)) + c_1 \sin(2\ln(x)) - \frac{1}{4}\cos(2\ln(x))\ln(x)$$

6) *Section 2.6 Problems* 63. (p.97)

$(x^2 - 2x)y'' + 2(1-x)y' + 2y = 0$

$y_1(x) = x^2$, $y_2(x) = x - 1$

Solve $(x^2 - 2x)y'' + 2(1-x)y' + 2y = 6(x^2 - 2x)^2$

➔ $$y'' + 2\frac{1-x}{x^2 - 2x} y' + \frac{2y}{x^2 - 2x} y = 6(x^2 - 2x)$$

$$y_p = u(x)y_1(x) + v(x)y_2(x)$$

$$u' = -\frac{y_2(x) f(x)}{W(x)}$$

$$v' = \frac{y_1(x) f(x)}{W(x)}$$

$$f(x) = 6(x^2 - 2x))$$

➔ $$W(x) = \begin{vmatrix} x^2 & x-1 \\ 2x & 1 \end{vmatrix} = x^2 - 2x^2 + 2x = -x^2 + 2x$$

==> $u(x) = 3x^2 - 6x$, $v(x) = -2x^3$

$$y_p = u(x)y_1(x) + v(x)y_2(x) = (3x^2 - 6x)x^2 + (-2x^3)(x-1)$$
$$= 3x^4 - 6x^3 - 2x^4 + 2x^3 = x^4 - 4x^3$$

➔ $y = c_1 x^2 + c_2(x-1) + x^4 - 4x^3$