Engineering Mathematics I HW#7 日河工 2B

1) *Sections 2.3 Problems* 14. a) (p. 76)

$xy'' = 2 + y'$

Set $u = y'$

➔ $xu' = 2 + u$, $u' - \frac{1}{x}u = \frac{2}{x}$

integrating factor $e^{\int -\frac{1}{x}dx} = e^{-\ln|x|} = \frac{1}{x}$

➔ $\frac{1}{x}u' - \frac{1}{x^2}u = \frac{2}{x^2}$, $\left(\frac{1}{x}u\right)' = \frac{2}{x^2}$

➔ $\frac{1}{x}u = \int \frac{2}{x^2}dx + c$, $u = -2 + cx$

$y = \int (-2 + cx)dx = -2x + \frac{1}{2}cx^2 + c_2 = -2x + c_1x^2 + c_2$

2) *Additional Problems* 19. (p. 112)

$x^2y'' - xy' - 2y = x^3 + 4\ln(x)$, $y(1) = 9$, $y'(1) = 7$

Let $x = e^t$

➔ $Y(t) = y(e^t)$, $Y'(t) = xy'(x)$, $x^2y'' = Y''(t) - Y'(t)$

➔ $Y'' - Y' - Y' - 2Y = e^{3t} + 4\ln(e^t) = e^{3t} + 4t$

$\lambda^2 - 2\lambda - 2 = 0$, $\lambda = 1 \pm \sqrt{3}$

$Y_h = c_1e^{(1+\sqrt{3})t} + c_2e^{(1-\sqrt{3})t}$

Set $Y_p = ae^{3t} + bt + c$

➔ $Y_p' = 3ae^{3t} + b$, $Y_p'' = 9ae^{3t}$

$9ae^{3t} - 6ae^{3t} - 2b - 2ae^{3t} - 2bt - 2c = e^{3t} + 4t$, ==> $ae^{3t} - 2bt - 2b - 2c = e^{3t} + 4t$

➔ $a = 1$, $b = -2$, $c = 2$

$Y_p = e^{3t} - 2t + 2$

$Y = Y_h + Y_p = c_1 e^{(1+\sqrt{3})t} + c_2 e^{(1-\sqrt{3})t} + e^{3t} - 2t + 2$

$\Longrightarrow y = c_1 e^{(1+\sqrt{3})\ln(x)} + c_2 e^{(1-\sqrt{3})\ln(x)} + e^{3\ln(x)} - 2\ln(x) + 2$

$\quad = c_1 x^{1+\sqrt{3}} + c_2 x^{1-\sqrt{3}} + x^3 - 2\ln(x) + 2$

$\Longrightarrow y(1) = c_1 + c_2 + 1 + 2 = 9, \quad y'(1) = c_1(1+\sqrt{3}) + c_2(1-\sqrt{3}) + 3 - 2 = 7$

$\Longrightarrow c_1 = c_2 = 3$

3) *Additional Problems* 20. (p. 112)

$y'' + y = \sec^3(x), \quad y(0) = 4, \quad y'(0) = 2$

➔ $\lambda^2 + 1 = 0, \quad \lambda = i, \quad -i$ ➔ $y_h = c_1\cos(x) + c_2\sin(x)$

$y_p = u(x)y_1 + v(x)y_2 = u\cos(x) + v\sin(x)$

$$W(x) = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) + \sin^2(x) = 1$$

$$u'(x) = -\frac{y_2 f(x)}{W(x)} = -\sin(x)\sec^3(x), \quad \Longrightarrow u = -\frac{1}{2\cos^2(x)}$$

$$v'(x) = \frac{y_1 f(x)}{W(x)} = \cos(x)\sec^3(x) = \sec^2(x), \quad \Longrightarrow v = \tan(x)$$

$$y_p = u\cos(x) + v\sin(x) = -\frac{1}{2\cos(x)} + \tan(x)\sin(x) = \frac{\tan(x)\sin(x)}{2}$$

➔ $y = c_1\cos(x) + c_2\sin(x) + \dfrac{\tan(x)\sin(x)}{2}$

$y(0) = c_1 = 4, \quad y'(0) = c_2 = 2$

$$y = 4\cos(x) + 2\sin(x) + \frac{\tan(x)\sin(x)}{2}$$

4) *Section 4.1 Problems* 1. (p. 168)

$y'' + y' - xy = 0, \quad y(0) = -2, \quad y'(0) = 0$

➔ $$y(x) = \sum_{n=0}^{\infty} \frac{1}{n!} y^{(n)}(0)x^n = y(0) + y'(0)x + \frac{1}{2}y''(0)x^2 + \frac{1}{6}y^{(3)}x^3 + \frac{1}{24}y^{(4)}(0)x^4 + \ldots$$

$$= -2 + \frac{1}{2}y''(0)x^2 + \frac{1}{6}y^{(3)}x^3 + \frac{1}{24}y^{(4)}(0)x^4 + \ldots$$

$y''(0) + y'(0) - 0 \cdot y(0) = 0, \quad \Longrightarrow y''(0) = 0$

$y''' + y'' - xy' - y = 0, \quad y^{(3)}(0) + y''(0) - 0 \cdot y'(0) - y(0) = 0, \quad y^{(3)}(0) = -2$

$y'''' + y''' - xy'' - y' - y' = 0, \quad y^{(4)}(0) + y^{(3)}(0) = 0, \quad y^{(4)}(0) = 2$

$y^{(5)} + y^{(4)} - y^{(2)} - xy^{(3)} - 2y^{(2)} = 0, \quad y^{(5)}(0) + y^{(4)}(0) = 0, \quad y^{(5)}(0) = -2$

$$y^{(6)} + y^{(5)} - y^{(3)} - xy^{(4)} - y^{(3)} - 2y^{(3)} = 0, \quad y^{(6)}(0) + y^{(5)}(0) - 4y^{(3)}(0) = 0, \quad y^{(6)}(0) = -6$$

➔ $y(x) = -2 - \frac{1}{3}x^3 + \frac{1}{12}x^4 - \frac{1}{60}x^5 - \frac{1}{120}x^6 + ...$

5) *Section 4.2 Problems* 1. (p.174)

$$y' - xy = 1 - x$$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

➔ $\sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^{n+1} = 1 - x, \quad ==> \sum_{n=0}^{\infty} (n+1)a_{n+1} x^n - \sum_{n=1}^{\infty} a_{n-1} x^n = 1 - x$

$$a_1 + \sum_{n=1}^{\infty} \left[(n+1)a_{n+1} - a_{n-1}\right]x^n = a_1 + (2a_2 - a_0)x + \sum_{n=2}^{\infty} \left[(n+1)a_{n+1} - a_{n-1}\right]x^n = 1 - x$$

➔ $a_1 = 1, \quad 2a_2 - a_0 = -1, \quad (n+1)a_{n+1} = a_{n-1} \qquad n \geq 2$

$$a_0 = C + 1, \quad a_1 = 1, \quad a_2 = C/2, \quad a_3 = 1/3, \quad a_4 = C/(2 \cdot 4), \quad ...$$

$$==> y = 1 + \sum_{n=0}^{\infty} \frac{1}{1 \cdot 3 \cdot 5 \cdots (2n+1)} x^{2n+1} + C\left(1 + \sum_{n=1}^{\infty} \frac{1}{2 \cdot 4 \cdot 6 \cdots 2n} x^{2n}\right)$$

or

$$==> y = C + 1 + C\left(\sum_{n=1}^{\infty} \frac{1}{2 \cdot 4 \cdot 6 \cdots 2n} x^{2n}\right) + \sum_{n=0}^{\infty} \frac{1}{1 \cdot 3 \cdot 5 \cdots (2n+1)} x^{2n+1}$$

$$= a_0 + (a_0 - 1)\left[\frac{1}{2}x^2 + \frac{1}{2(4)}x^4 + \frac{1}{2(4)(6)}x^6 + \frac{1}{2(4)(6)(8)}x^8 + ...\right] +$$

$$x + \frac{1}{3}x^3 + \frac{1}{3(5)}x^5 + \frac{1}{3(5)(7)}x^7 + \frac{1}{3(5)(7)(9)}x^9 + ...$$