海大河工系工數二(B)第三次大考參考解答(拉氏轉換)

1. Solve the complementary solutions of $y'''=0$ using Laplace transform. (5%)

   Ans： $y(t)=c_1+c_2t+c_3t^2$

   Solve the solutions of $y'''-3y''+3y'-y=0$ using Laplace transform (5%)

   Ans： $y(t)=c_1e^t+c_2te^t+c_3t^2e^t$

2. Solve the total solution of $y''+y=\sin(t)$ subject to $y(0)=0$ and $y'(0)=0$ using Laplace transform (10%)

   Ans： $y(t)=\frac{-t}{2}\cos t+\frac{1}{2}\sin t$

3. Find the Laplace transform of e^t, cosh(t), cos(t) and $d(t)$.(10%)

   Ans： $e^t\rightarrow\frac{1}{s-1}$, $\cosh(t)\rightarrow\frac{s}{s^2-1}$, $\cos(t)\rightarrow\frac{s}{s^2+1}$, $d(t)\rightarrow 1$

4. Take Laplace transform of $t^2y''-2ty'-10y=0$. (5%)

   Ans： $s^2Y''(s)+6sY'(s)-6Y(s)=0$

5. Take Laplace transform of $t^2y''+6ty'-6y=0$. (5%)

   Ans： $s^2Y''(s)-2sY'(s)-10Y(s)=0$

6. Solve the solution of $y'+y=\sin(t)+\cos(t)$ subject to y(0)=0. (10%)

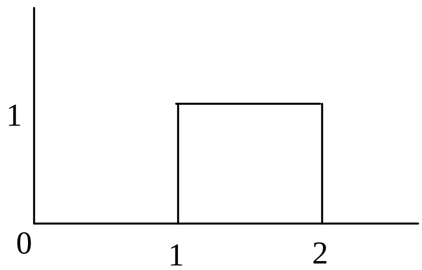
   Ans： $Y(s)=\frac{1}{s^2+1}$, $y(t)=\sin t$

7. What is convolution (5%) ?

   Ans: $f(t)*g(t)=\int_0^t f(t-t)g(t)dt$

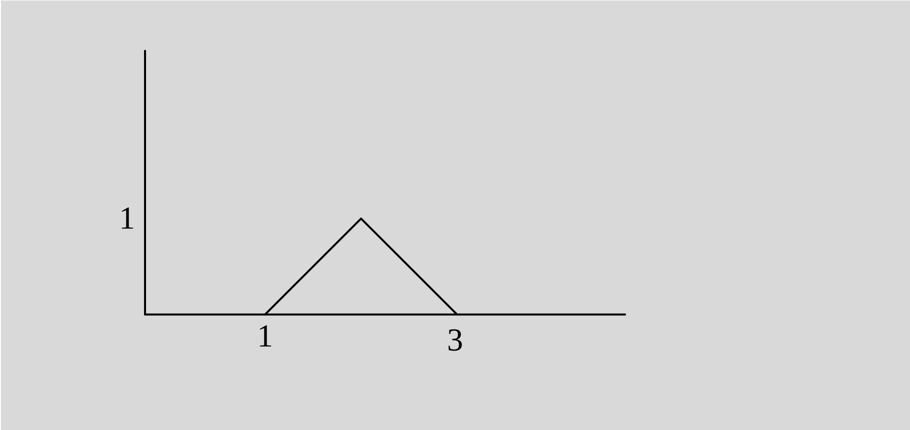
8. Plot a(t)=U(t)-U(t-1) and b(t)=U(t-1)-U(t-2) where U(t) is shown below. (10%)

a(t) b(t)



9. Find the convolution of a(t) and b(t), c(t)=a(t)*b(t) (15%)

   Ans: $c(t)=\begin{cases} t-1 & ,1<t<2 \\ 3-t & ,2<t<3 \end{cases}$



10. Find the Laplace transform of U(t) (2%) and c(t). (8%)

Ans： $L\{U(t)\}=\frac{1}{s}$, $L\{c(t)\}=\frac{1}{s^2}e^{-s}(1-e^{-s})^2$

11. Please write the initial value theorem and final value theorem. (10%)

初值定理: $\lim_{t\to 0} f(t)=\lim_{s\to\infty} sF(s)$

終值定理: $\lim_{t\to\infty} f(t)=\lim_{s\to 0} sF(s)$

12. If the Laplace transform of $1/\sqrt{t}$ is P(s), find the Laplace transform of $\sqrt{t}$ in terms of P(s). (10%)

(Hint: two choices: differential operator and multiplying by t)

U(t)=1 t>0 otherwise U(t)=0

Ans: differential operator: $-P'(s)$

multiplying: $\frac{1}{2s}P(s)$