

Mid-term Exam I

Nov. 2004

1) Verify the given function is a solution of the differential equation (10 scores)

a) $y' = -\frac{2y + e^x}{2x}$ for $x > 0$; $\varphi(x) = \frac{C - e^x}{2x}$

b) $y' = y^2 e^{-x}$, $\varphi(x) = \frac{1}{e^{-x} - k}$

2) Verify by implicit differentiation that the given equation implicitly defines a solution of the differential equation (5 scores)

$$y^2 + xy - 2x^2 - 3x - 2y = C; \quad y - 4x - 3 + (x + 2y - 2)y' = 0$$

3) Consider $y' = \frac{y}{x} + 1$ for all $x > 0$ (20 scores)

a) get the particular solution corresponding to the initial solution $y(1)=0$ (10 scores)

b) draw a direction field of the differential equation and the integral curve through $(1, 0)$ *hint: $\ln(2) \cong 0.69$* (10 scores)

4) Given a family F of curves $x^2 - Ky^2 = 1$ (25 scores)

a) describe the definition of orthogonal trajectories of a given family of curves (5 scores)

b) find the family of orthogonal trajectories of the given family F of curves (15 scores)

c) plot the orthogonal families together on the x-y plane (5 scores)

5) Solve $y' + y/x = \cos(x)$, $x > 0$ (15 scores)

6) Solve $x - xy^2 - y' = 0$ (15 scores)

7) Solve $y' = \frac{y}{x-y}$ (10 scores)