Mid-term Exam II

Dec. 2004

1) Consider $y' = \frac{y}{x} + 1$ for all $x > 0$

a) get the particular solution corresponding to the initial solution y(1)=0 (5 scores)

b) draw the integral curve through (1, 0) *hint*: $\ln(2) \cong 0.69$ (5 scores)

2) Consider $y' = \frac{y}{x - y}$

a) find the general solution (5 scores)

b) Verify your answer is the general solution (3 scores)

c) Does the general solution contain all the solutions ? If not, point out the singular solution. (2 scores)

3) Find a differential equation having the given function as general solution

a) $c_1 e^{-2x} + c_2 e^{3x}$ (2 scores)

b) $e^{-x}[c_1 \cos(\sqrt{5}x) + c_2 \sin(\sqrt{5}x)]$ (2 scores)

c) $e^{3x}(c_1 + c_2 x)$ (2 scores)

d) $c_1 x^3 + c_2 x^3 \ln(x)$ (2 scores)

e) $c_1 x^3 \cos(\ln(x)) + c_2 x^3 \sin(\ln(x))$ (2 scores)

4) Consider $y'' + 2y' + y = -3e^{-x} + 8xe^{-x} + 1$

a) find y_1, y_2 to form a fundamental set of solutions of the homogeneous equation $y'' + 2y' + y = 0$ (5 scores)

b) show that y_1, y_2 are linearly independent (5 scores)

c) find the general solution of $y'' + 2y' + y = 0$ (2 scores)

d) find the particular solution of $y'' + 2y' + y = -3e^{-x} + 8xe^{-x} + 1$ (10 scores)

e) write the general solution of $y'' + 2y' + y = -3e^{-x} + 8xe^{-x} + 1$ (3 scores)

5) Consider the initial value problem $y'' + y = \sec^3(x)$; $y(0) = 4$, $y'(0) = 2$

a) find y_1, y_2 to form a fundamental set of solutions of the homogeneous equation $y'' + y = 0$ (5 scores)

b) find the particular solution of $y'' + y = \sec^3(x)$ (10 scores)

c) find the particular solution of $y'' + y = \sec^3(x)$; $y(0) = 4$, $y'(0) = 2$ (5 scores)

6) Solve $(x^2 + 1)y'' - 2xy' + 2y = (x^2 + 1)^2$ with $y_1(x) = 1 - x^2$ (10 scores)

7) Find the first five terms of the power series solution of $y'' + xy' - y = e^{3x}$ (15 scores)