# QUIZ-3th　　日河工 2B

Nov. 3, 2004

1) Solve the differential equation $3y' = 4x / y^2$ (15 scores)

➔ $3y^2 dy = 4x dx$, the differential equation is separable.

By direct integration, $\int 3y^2 dy = \int 4x dx$ ➔ $y^3 = 2x^2 + C$

2) Solve the differential equation $\frac{2xy}{y-1} - y' = 0$ (15 scores)

➔ $\frac{y-1}{y} dy = 2x dx, y \neq 0$, $\int \left(1 - \frac{1}{y}\right) dy = \int 2x dx$, $y - \ln|y| = x^2 + C$

The general solution is $y - \ln|y| = x^2 + C$

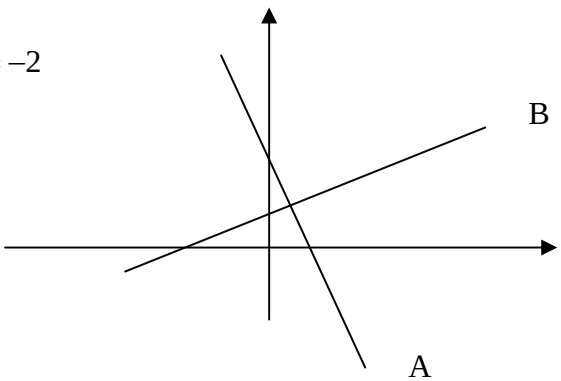
But $y = 0$ is still a solution of the DE (try it by substitution), though it cannot be contained in the expression for the general solution for any choice of $C$.

3) a)What is the slope of line A $y = x/2 + 1$ (5 scores) ? plot the line A in x-y plane (5 scores)

slope = $y' = \frac{dy}{dx} = 1/2$

b)What is the slope of line B $y = -2x + 2$ (5 scores) ? plot the line B in x-y plane(5 scores)

slope = $y' = \frac{dy}{dx} = -2$



c)Consider the family *F* of curves that are graphs of $F(x,y,K)=y-Kx^2=0$

Find the family *G* of orthogonal trajectories of the family *F* of curves (20 scores)

*See p.56~57 of the textbook*

4)a)Verify that $y_1(x)=e^{-3x}$, $y_2(x)=e^{-8x}$ are solutions of the differential equation $y''+11y'+24y=0$ (10 scores)

b)Show that their Wronskian is not zero (10 scores)

c)Write the general solution of the differential equation (5 scores)

d)Find the solution of the initial value problem with $y(0)=1$, $y'(0)=4$ (10 scores)

a) $y_1(x)=e^{-3x}$, $y_2(x)=e^{-8x}$

$y_1'(x)=-3e^{-3x}$, $y_2'(x)=-8e^{-8x}$

$y_1''(x)=9e^{-3x}$, $y_2'(x)=64e^{-8x}$

➔ $y_1''+11y_1'+24y_1=9e^{-3x}+11(-3e^{-3x})+24e^{-3x}=(9-33+24)e^{-3x}=0$

$y_2''+11y_2'+24y_1=64e^{-8x}+11(-8e^{-8x})+24e^{-8x}=(64-88+24)e^{-8x}=0$

b) $W(x)=y_1y_2'-y_1'y_2=e^{-3x}(-8e^{-8x})-(-3e^{-3x})e^{-8x}=-5e^{-11x}\neq 0$

c) $\because W(x)\neq 0$ ➔ $y_1$, $y_2$ are linearly independent

➔the general solution is $c_1y_1+c_2y_2$ with $c_1$, $c_2$ arbitrarily constants.

d) $y(0)=c_1y_1(0)+c_2y_2(0)=c_1e^0+c_2e^0=c_1+c_2=1$

$y'(0)=c_1y_1'(0)+c_2y_2'(0)=c_1(-3e^0)+c_2(-8e^0)=-3c_1-8c_2=4$

➔ $c_1=12/5$, $c_2=-7/5$ ➔ $y=12y_1/5-7y_2/5=12e^{-3x}/5-7e^{-8x}/5$

5) a)Verify that $y_1(x)=\cos(x)$, $y_2(x)=\sin(x)$ are solutions of the differential equation $y''+y=0$ (10 scores)

b)Show that their Wronskian is not zero (10 scores)

c)Write the general solution of the differential equation (5 scores)

*See example 2.1, 2.2 of the textbook.*