QUIZ-4th　　　　日河工 2B

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1) Verify by implicit differentiation that the given equation implicitly defines a solution of the differential equation (20 scores)

$y^2 + xy - 2x^2 - 3x - 2y = C$; $y - 4x - 3 + (x + 2y - 2)y' = 0$

$d(y^2 + xy - 2x^2 - 3x - 2y)/dx = d(C)/dx$

==> $2yy' + y + xy' - 4x - 3 - 2y' = 0$

==> $y - 4x - 3 + xy' + 2yy' - 2y' = 0$

==> $y - 4x - 3 + (x + 2y - 2)y' = 0$

2) Consider $y' = \frac{y}{x} + 1$ for all $x > 0$ (80 scores)

a) get the particular solution corresponding to the initial solution y(1)=0 (30 scores)

b) draw a direction field of the differential equation and the integral curve through (1, 0) *hint*: $\ln(2) \cong 0.69$ (50 scores)

a) $y' = \frac{y}{x} + 1$ ==> $y' - \frac{y}{x} = 1$

➔ $\mu(x) = e^{\int -\frac{1}{x}dx} = e^{\ln(1/x)} = \frac{1}{x}$

➔ $\frac{y'}{x} - \frac{y}{x^2} = \frac{1}{x}$, $\left(\frac{y}{x}\right)' = \frac{1}{x}$, $\frac{y}{x} = \int \frac{1}{x}dx = \ln(x) + C$

➔ $y(x) = x\ln(x) + Cx$, $y(1) = \ln(1) + C = 0$, $C = 0$

➔ $y(x) = x\ln(x)$

b) <u>1st Step</u>---**for the direction field**

y ' =f(x,y)=constant ***curves of equal inclination***

as shown in the figure, you can set y ' =f(x,y)=1,1.5,2,2.5,3,3.5,4,5,7, respectively.

Namely, you first plot the ***curve of equal inclination*** $y' = \frac{y}{x} + 1 = 2$, $\Rightarrow y = x$

<u>2nd Step</u>

Along each curve f(x,y)=C, ***draw lineal elements indicating the prescribed slopes (i.e.*** above-mentioned 1,1.5,2,2.5,3,3.5,4,5,7 ***) ==>direction field***

The **integral curve** through (1, 0) means the curve of $y(x) = x\ln(x)$.