

# QUIZ-5<sup>th</sup>

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1) Solve the differential equation  $y'' + y = 0$  for all  $x$  and perform the Wronskian test

Set  $y = e^{\lambda x}$

$$\rightarrow \lambda^2 + 1 = 0, \quad \lambda = \pm i$$

$$\rightarrow y_1(x) = \cos(x), \quad y_2(x) = \sin(x)$$

$$W(x) = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) + \sin^2(x) = 1 \neq 0$$

$$\rightarrow y = c_1 \cos(x) + c_2 \sin(x)$$

2) Solve the initial value problem  $x^2 y'' - 5xy' + 10y = 0$ ;  $y(1) = 4$ ,  $y'(1) = -6$

Set  $t = \ln(x)$

$$\rightarrow Y'' - 6Y' + 10Y = 0, \quad Y(t) = c_1 e^{3t} \cos(t) + c_2 e^{3t} \sin(t)$$

$$\rightarrow y(x) = c_1 x^3 \cos(\ln(x)) + c_2 x^3 \sin(\ln(x))$$

$$\text{To meet } y(1) = 4 \rightarrow y(1) = c_1 = 4$$

$$\rightarrow y'(x) = 12x^2 \cos(\ln(x)) - 4x^2 \sin(\ln(x)) + 3c_2 x^2 \sin(\ln(x)) + c_2 x^2 \cos(\ln(x))$$

$$\text{To meet } y'(1) = -6 \rightarrow y'(1) = 12 + c_2 = -6, \quad c_2 = -18$$

3) Solve  $y'' - 6y' + 9y = 5e^{3x}$

Set  $y = e^{\lambda x}$

$$\rightarrow \lambda^2 - 6\lambda + 9 = 0, \quad \lambda = 3, \quad 3$$

$$\rightarrow y_h(x) = c_1 e^{3x} + c_2 x e^{3x}$$

$$\rightarrow y_p(x) = ax^2 e^{3x} \rightarrow a = 5/2$$

$$\rightarrow y(x) = c_1 e^{3x} + c_2 x e^{3x} + \frac{5}{2} x^2 e^{3x}$$